The Lorentz contraction

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Introduction

The contraction was proposed Fitzgerald as a simple way of accounting for the null result of the Michelson Morley experiment. Lorentz was the first to explain why it should occur and our unified theory broadly follows his ideas. However, his understanding of the effects of the contraction was somewhat incomplete.

The contraction is one of three effects which have been observed experimentally and are well known; the other two being the increase in inertial mass and the slowing of time dependant processes. The forth effect is less obvious, but equally well evidenced. The effect on time dependant processes extends to the atomic clock and with the advent of the GPS system is the most accurately measured. But atomic clocks count the beat frequency between two spectral lines of the hyperfine structure of cesium. These frequencies depend on energy levels within the atoms, so the slowing of an atomic clock directly measures a change in the energy levels within atoms and reveals a reduction in energy levels as the fourth effect. The result of this is that energy measured within a moving system appears greater than it really is.

In our unified theory, we describe how nature works through simple causal processes. Matter is composed of elementary charged particles whose electric fields coexist in space forming a background against which the motion of electric flux generates magnetic intensity and the motion of magnetic flux generates electric intensity. These two actions, described by Maxwell's Laws, make it possible for photons and radio waves to exist and travel through the background at the velocity of light. These same actions also affects the electric fields of elementary charged particles. The first is responsible for the generation of a magnetic field which contains the particle's kinetic energy. The second is responsible for the Lorentz contraction which is in turn responsible for the other effects. There is an order of causation:

Maxwell's Laws	$\Rightarrow$	Lorentz contraction
Lorentz contraction	$\Rightarrow$	Increase in inertial mass and decrease in Einstein mass
Slowing of clocks	$\Rightarrow$	Changes in mass
Synchronisation errors	$\Rightarrow$	Slowing of clocks

Changes in mass

In our unified theory, we distinguish between different manifestations of mass because while it may be a simple concept in Classical Physics, when we try to take into account the effects of near light speed and gravitational potential, things become somewhat more complicated. Newton identified three properties of mass: inertial mass, passive gravitational mass and active gravitational mass and showed that they are equal. (In fact they are proportional and set equal by the way we define our units.) Einstein introduced the concept that mass and energy are equivalent with his equation $E = mc^2$ giving yet another kind of mass which in unified theory we refer to as Einstein mass. The relativistic doppler effect and the slowing of atomic clocks reveal that the energy levels have been reduced by a factor γ, so the Einstein mass calculated as $m_E = \frac{E}{c^2}$ is reduced by a factor γ. The mass increase observed when acclerating charged particles to near light speed is a combination of the two effects. For acceleration in the direction of motion, the observed $m_l = \gamma^3 m_0$ results from a decrease in the Einstein mass by a factor of γ and an increase in the inertial action of the Einstein mass and there is a simmilar effect for acceleration perpendicular to the direction of motion.

$$m_l = \gamma^4 m_E = \gamma^4 \left(\frac{1}{\gamma} m_0 \right) = \gamma^3 m_0 \qquad m_t = \gamma^2 m_E = \gamma^2 \left(\frac{1}{\gamma} m_0 \right) = \gamma m_0$$

Maths proof

Lorentz identified two of the fundamental equations of electricity and magnetism as being special cases of the same equation[3iii]:

Maxwell's wave equation $\nabla^2 \varphi - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \varphi = 0$ and Poisson's equation $\nabla^2 \varphi = \frac{-\rho}{\varepsilon_0}$

are special cases of $\nabla^2 \varphi - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \varphi = \frac{-\rho}{\varepsilon_0}$

Now in the case of the electric field of a moving electron, or the electric fields within a moving system of charged particles, $\frac{\partial^2}{\partial t^2} = \frac{dx}{dt} \frac{\partial}{\partial x} \left(\frac{dx}{dt} \frac{\partial}{\partial x} \right) = v^2 \frac{\partial^2}{\partial x^2}$. Upon expanding $\nabla^2 \varphi$ and collecting terms:

$$\left(1 - \frac{v^2}{c^2} \right) \frac{\partial^2}{\partial x^2} \varphi + \frac{\partial^2}{\partial y^2} \varphi + \frac{\partial^2}{\partial z^2} \varphi = \frac{-\rho}{\varepsilon_0}$$

Making the substitution $x = \sqrt{1 - \frac{v^2}{c^2}}\, x'$ $\quad y = y'$ $\quad z = z'$

$$\left(1 - \frac{v^2}{c^2} \right) \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial x'^2} \quad : \quad \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial y'^2} \quad : \quad \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial z'^2}$$

reduces the equation to $\frac{\partial^2}{\partial x'^2} \varphi + \frac{\partial^2}{\partial y'^2} \varphi + \frac{\partial^2}{\partial z'^2} \varphi = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \quad \nabla^2 \varphi = \frac{-\rho}{\varepsilon_0} \; in\, x'y'z'$

This is interpreted as meaning that if we have a system, held in equilibrium by electrostatic forces when at rest and described by co-ordinates *x*, *y*, and *z*, that, when the system is in motion, its condition of equilibrium is governed by the same equations written in the new co-ordinates *x′*, *y′*, and *z′*.

A distance $\delta x'$ measured in the moving system remains the same as it would be when the system was at rest in the stationary system because the ruler we use to measure it has also suffered a contraction. If we had some god-given ruler which was not affected by motion through the stationary system it would measure the distance to be shorter:

$$\partial x = \sqrt{1 - \frac{v^2}{c^2}}\, \partial x'$$

The result is that the surface of the electron and its electric field as described by $\vec{D}$ and ϕ is Lorentz contracted. This is a real contraction caused by a real velocity through the background.

It is customary to use either the symbol β or γ defined as:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Sometimes the factor γ is involved in an increase as in mass; sometimes in a decrease as in length. Since γ is always bigger than 1, it acts either as a multiplier or divisor . When we say something is increased by a factor γ, we mean multiplied by γ. When we say it is decreased by a factor γ, we mean that it is divided by γ.

Effects of the contraction

Lorentz's proof involves the electric field as described by its flux density and electric potential. The increase in electric flux density increases the flux density of the magnetic field generated by the electron's motion and this increases its kinetic energy. Since acceleration involves doing work to impart kinetic energy, more work must be done to accelerate the electron and this requires a greater force to achieve the same rate of acceleration. The result is that the mass seems to have increased. Experimental results suggested that the inertial mass resisting linear acceleration is different from that resisting centripetal acceleration. This led respectively to the terms "longitudinal mass" m_l resisting acceleration in the direction of motion and "transverse mass" m_t resisting acceleration perpendicular to the direction of motion. They are found by experiment to be:

$$m_l = \gamma^3 m_0 \qquad m_t = \gamma m_0$$

Interatomic spacing is determined by electric potential and found by solving Poisson's equation. So everything within a moving system suffers the Lorentz contraction. Mechanical and interatomic forces result from electric field strength acting on the charges of electrons and nuclei. However Electric potential is more fundamental, electric field strength being the gradient of the potential.

$$\vec{E} = \nabla\phi$$

The gradient is the rate of change of potential with distance and the contraction increases the component of the gradient which lies in the direction of motion. This means that within a moving system all forces in the direction of motion are increased by a factor γ.

Consider the acceleration of electrons in laboratory experiments. The velocity of the Laboratory through the background may be assumed to be so small compared with the speed of light that the laboratory is effectively the stationary system. We can then consider the charged particle as the moving system. In accelerating it, we are applying a force from the stationary system upon the moving system. To understand this more fully, let us consider the forces exerted on each other by an electron and a proton as the electron is accelerated towards the proton. Because of there relative masses, we may assume that the electron will reach near light speed and its potential field become contracted. Each is acted on by the gradient of the potential of the other. If they are a distance d apart, and $\phi_{e,P}$ is the potential due to the electron at the location of the proton, etc. then:

$$\phi_{e,P} = \frac{-e}{4\pi\,\varepsilon_0\,\gamma\,d} \qquad \phi_{P,e} = \frac{e}{4\pi\,\varepsilon_0\,d}$$

Differentiating to find the electric field strengths:

$$E_{e,P} = \frac{1}{\gamma}\frac{e}{4\pi\,\varepsilon_0\,d^2} \qquad E_{P,e} = \frac{e}{4\pi\,\varepsilon_0\,d^2} \qquad \Rightarrow \qquad E_{e,P} = \frac{1}{\gamma}E_{P,e}$$

And we see that the force experienced by the electron is γ times the force experienced by the proton. (Note the lost minus sign due to the fact that the directions from one to the other, with respect to which we differentiate, are opposite.)

Applying this to the acceleration of a charged particle in the laboratory, the force acting within the moving system is γ times the force applied from the stationary system. Now we are taught that forces come in equal and opposite pair, so this situation is most odd. The explanation is that the contraction creates a velocity ratio between the two systems which gives one a mechanical advantage over the other. This effect combines with the reduction in Einstein mass such that they compensate for each other. The effect on the Einstein mass is only noticable from the staionary system. Withint he moving system, we are unaware of it, so the mass appears to be γ times its real value and the accelerating force also appears to increased by a factor γ.

Since we define inertial mass as the ability to resist acceleration, it appears from within the moving system that mass is increase by a further factor of gamma giving:

$$m_l = \gamma^4 m_0 \qquad m_t = \gamma^2 m_0$$

This increase in mass within the moving system affects all time dependant processes including clocks. This is easy to see in the case of a harmonic oscillator where a restoring force $F = -\lambda x$ is proportional to the displacement *x*.

$$F = m a \qquad \Rightarrow \qquad \frac{d^2x}{dt^2} = -\frac{\lambda}{m} x \qquad \Rightarrow \qquad T = 2\pi\sqrt{\frac{\lambda}{m}}$$

The effect of the contraction within the moving system increases the component of force in the direction of motion, but distance is also decreased, so the modulus is increased by a factor of γ^2 for oscillations in the direction of motion giving:

$$T = 2\pi\sqrt{\frac{\gamma^2 m}{\lambda}} \qquad \perp \text{ direction of motion}$$

$$T = 2\pi\sqrt{\frac{\gamma^4 m}{\gamma^2 \lambda}} \qquad \parallel \text{ direction of motion}$$

We see that the period of oscillation is increased by a factor γ with the result that all clocks run slow by a factor γ.

The slowing of clocks also results in synchronisation errors. The first attempts to synchronise clocks across a country came with the railways. Before the railways, clocks were set from the sun and stars; so London was ahead of Bristol by 10 minutes. To make sense of their timetables, the railways established railway time by physically taking a clock by train down the line and setting all the station clocks from it. When a clock is transported within the moving system in the direction of motion of the system, it runs even more slowly and has lost time resulting in an error in the synchronising of clocks.

Atomic clocks

We have seen that it appears from within a moving system that masses and forces are increased by a further factor of γ, and despite the explanation of one system having a mechanical advantage over the other, the whole idea seems to lack credibility. However, the behaviour of atomic clocks sheds light on the situation.

An atomic clock works by counting a beat frequency between two hyperfine lines of the cesium spectrum. Since the beat frequency is proportional to the energy levels within th atom, any change in the rate of an atomic clock is a measure of a change in the energy levels within the atom. So when an atomic clock is taken upstairs and found to run slightly faster, the explanation is that the work done carrying it up stairs has increased the energy levels within its atoms making them less negative. The implication is that the slowing of an atomic clock on board a GPS satellite indicates that the contraction has resulted in a reduction in energy levels with its atoms making them more negative. This is a clue as to a further effect of the contraction.

In our initial unified theory, we made the mistake of following Lorentz in assuming that the reduction in the size of the volume element reduced the value of the integral. We now know this to be wrong. Lorentz's integrals are parametric integrals and remain valid. This leaves us with an inadequate explanation of the behaviour of the electric field and it would now seem that the energy content of the electric field is increased by a factor of γ. The standard model which reject denies that any energy is stored in any magnetic field and instead attributes all changes to the electric field. It is a one or the other situation. If energy is stored in the magnetic field generated by the motion of the particle, then the energy content of its electric field must be invariant.

To take account of the slowing of atomic clocks, we have to slightly modify our unified theory. We now assert that the energy content of the electric field is invariant simply because there is no mechanism to alter it with a change in velocity. The increase in velocity increases the energy density within the electric field and it follows that if the total energy is to remain constant, the only other parameter which can vary is the radius of the electron. Thus while distances within and between atoms are subject to the contraction, the actual electrons and quarks suffer an expansion by a factor of γ perpendicular to the direction of motion.

While their electric fields are measurable, the size of the core of an electron remains a mystery other than the fact that we calculate its size from our theory of electromagnetic mass. Within the moving system, we are completely unaware that any change has taken place. But it has, because when our moving GPS satellite broadcasts the time of its atomic clock, it reveals the fact that it running slow. The energy $h\nu$ of the photons has decreased. Since the plank constant is invariant, then our unit of energy within the moving system has reduced in magnitude.

When we consider the energy content to be $m\,c^2$, the mass increase of $m_l = \gamma^4 m_0$ and $m_t = \gamma^2\, m_0$ which we see within the moving system equate to the same increase in energy that we observe from the stationary system where it appears to us that $m_l = \gamma^3 m_0$ and $m_t = \gamma\, m_0$.