Lorentz transforms

The Lorentz transforms apply to observers moving in a state of uniform motion. They work when two observers moving relative to one another set up co-ordinate systems according to a set of seven rules:

- The points they chose as origin must at some moment be in the same place (coincident).
- They must use the line of sight from one origin to the other as their x axes.
- Their x axes must point in the same direction.
- Their y and z axes must appear parallel as seen looking along the x axes.
- They must each have a master clock at their origin.
- They must set their master clocks to zero when their origins are coincident. (This requires a little imagination because in reality, the clocks would collide.)
- They each use light pulses or radio signals between clocks to calibrate and synchronise local clocks spread around their co-ordinate grids.

The Lorentz transform takes into account the effects of motion on rulers and clocks transforming both the xyz position co-ordinates and the time of an event as recorded by a local clock.

If the co-ordinates in the stationary system are x, y, z and t and those of the moving system are x', y', z' and t'. The transform equations are:

$$x' = \frac{x - v\,t}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y' = y \qquad z' = z \qquad t' = \frac{t - \frac{v\,x}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The Lorentz transforms are linear transforms since the co-ordinate and time variables appear to the power of 1 in the equations. Linear transforms can be performed by matrix algebra and we might alternatively write:

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\,v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\frac{v}{c^2} & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \qquad : \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

[The reader may be expecting the bottom left element to be $-\gamma\,v$ because many authors use co-ordinates with units in which the speed of light is 1 resulting in a more symmetric matrix.]

Warning

The Lorentz transforms relate to two observers recording the position and time of an event and then comparing notes. They do not transform what one observer sees into the view seen by the other observer. This is because of the effects of the synchronisation errors. We see only the past. This only becomes significant when we look at distant objects because light travels at approximately 1 foot per nanosecond. We can calculate the effects of this, but we are moving through the background in which light travels at a constant speed and there is no way of knowing how fast and which direction we are moving. If we view an event 1000 ft away, we think it happened 1000 nanoseconds before we saw it. But if the earth is moving through the background at 1% of the speed of light, then the synchronisation error could be as much as 10 nanoseconds. If something is moving, these synchronisation errors will affect the way we see it.

Two observers in relative motion will see the world around them with different synchronisation errors. The Lorentz transforms consequently translate one observer's view of simultaneous events into a set of events spread through time in the other observer's system. A round ball in one system is turned into ellipsoid smeared through time in the other system. Looking at the co-ordinates, it is apparently elongated in the direction of motion because the front of it has a time co-ordinate latter than that of the rear, so has travelled further. When we correct the position/time co-ordinates so that they are all synchronised to give the view seen by the other observer, the the ball is now seen to be contracted in the direction of motion.

Just as one might say that a fundamental principle of mechanics is that forces come in equal and opposite pairs, so we discover that effects in relativity often come in reciprocal pairs requiring us to take great care to be sure which one applies to a given situation. One member of the alt.physics.relativity community stated "It is impossible to derive an answer in relativity without knowing the answer first." While this is an exaggeration, relativity requires a great deal of clear thinking and it is all too easy to give the wrong reasons for the right answer.

Which brings us back to Einstein who had read the papers of Lorentz and Poincaré and consequently knew the right answers. But he could not simply copy their papers. If he was to pass the theory of as his own original work, he needed a different set of reasons for producing the right answer. Once Einstein's alternative reasons became part of the doctrine taught to students, clear thinking became very difficult for subsequent generations.