

Cyclotron Motion

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The cyclotron was an early form of particle accelerator which relied on a rather neat electromagnetic property. We a charge q moving with velocity \vec{v} through a magnetic field of flux density $\vec{B} = \mu_0 \vec{H}_0$ experiences a force $\vec{F} = \mu_0 q \vec{v} \times \vec{H}_0$ which is perpendicular to its direction of motion. This will cause it to move in an arc generating an equal and opposite centrifugal force $m \frac{v^2}{r}$. When $\vec{v} \perp \vec{H}_0$ the charge will describe a circle of radius r . When we equate the forces we find that the charge moves with constant angular velocity $\omega = \frac{v}{r}$:

$$\mu_0 q v H_0 = \frac{m v^2}{r} \Rightarrow \omega = \frac{v}{r} = \frac{q}{m} \mu_0 H_0; \quad r = \frac{m v}{\mu_0 q H_0}$$

The charge takes the same time to complete a circle regardless of its velocity. The cyclotron exploits this by surrounding the path with two semicircular electrodes and applying an AC voltage between them such that as the electrons orbit they are accelerated by the voltage between the electrodes as they pass through the gap between them. Charged particles are introduced near the centre and move outwards every half cycle as they are accelerated.

If the magnetic field is strong enough and the velocity small enough the current loop formed by the charge will have a great enough magnetic moment to form its own magnetic field with $n \Phi_0$ quanta of magnetic flux threading the orbit. If the radius and velocity are comparable with an electron orbiting within an atom, we can assume that magnetic field contains half of the kinetic energy of the charges and obeys the same equation as the electron of hydrogen atom:

$$\frac{1}{2} v q n \Phi_0 = \frac{1}{2} 2 m \pi^2 r^2 v^2$$

We can solve for v in terms of r and substitute in the various equations and expressions to give:

$$v = \frac{q n \Phi_0}{2 m \pi^2 r^2} \Rightarrow$$
$$L = 2\pi r^2 v m = 2\pi r^2 \frac{n q \Phi_0}{2 \pi^2 m r^2} m = \frac{n q \Phi_0}{\pi} = n \frac{q}{e} \hbar$$
$$\mu = I A = q v \pi r^2 = q \frac{n q \Phi_0}{2 \pi^2 m r^2} \pi r^2 = \frac{n q^2 \Phi_0}{2\pi m} = \frac{q^2}{2e m} n \hbar$$

So a charged particle in cyclotron motion may under the right conditions display quantised behaviour. Indeed, we might well expect this quantised behaviour to be associated with adsorption and emissions of photons. If one did not know what was going on, one might wrongly assume that the particle had an intrinsic magnetic moment which was coupling with the field. It will come as no surprise that this has been discovered.

Electrons in a Penning trap exhibit the same quantised behaviour as electrons in atoms. This was discovered after the author graduated! So, following the career of a maths teacher, he remained ignorant of this until recently! The discovery of this behaviour of electrons could have put an end to his meddling in quantum physics, but for one lucky chance. In the previous week, the author had solved the problem of the proton structure discovering that the quantised behaviour is independent of the nature of the force constraining the charged particle to move in a circle.