Strong gravitational fields

HOME: The Physics of Bruce Harvey

We will regard a strong gravitational field as one in which $e^{\frac{\Phi}{c^2}}$ is significantly less than 1. The length of rulers is significantly reduced. If we could use rulers to measure the circumference and radius of an orbit, we would find that the radius was greater than it should be: $r > \frac{C}{2\pi}$. We might venture out in space to a very large orbit where the effect of gravitational potential is negligible, measure its circumference and calculate its radius. If we now measure back from this to a smaller orbit and subtract, we will get yet another value for the radius. This raises the question as to how we should calculate the gravitational potential. The results we have so far obtained regarding the effects of gravitational potential have been independent of the fact that potential is a function of radius. We did not consider moving a mass through a height, but rather from one potential to another.

To gain a better understanding of the effects of gravitational potential, let us consider a system in which a planet is in a highly eccentric orbit and has a moon and that the relative size of the orbits is such that the gravitational potential Φ_s of its sun may be considered uniform over the planet moon system. We can look at the planet moon system as moving into regions of different background gravitational potential. This is no different from the way our own solar system is moving through the background gravitational potential field Φ_g of our galaxy. Potential is additive. That means that the effect of gravitational potential is multiplicative:

$$e^{\frac{\Phi_s + \Phi_g}{c^2}} = e^{\frac{\Phi_s}{c^2}} e^{\frac{\Phi_g}{c^2}}$$

Now the radius of the moon's orbit has been calculated locally by equating the force of gravity with centripetal force.

$$\frac{G\,M\,m}{r^2} = F = m\,\omega^2\,r$$

If we now take into account the gravitational potential of the sun, each of the local measurements in the equation must be multiplied by an appropriate power of $e^{\frac{\Phi_s}{c^2}}$. The right hand side is easy to analyse.

$$RHS \qquad m_i\,\omega^2\,r \rightarrow e^{-3\frac{\Phi_s}{c^2}} m_i \times \left(e^{\frac{\Phi_s}{c^2}} \omega\right)^2 \times e^{\frac{\Phi_s}{c^2}} r$$

The left hand side needs interpretation. If we look at the dimensional analysis results, G would appear to be affected as $e^{8\frac{\Phi_s}{c^2}}$ which is so ridiculous that we have not included it in the table. This is because dimensional analysis treats all mass as inertial mass. If we define G as a universal constant, then the gravitational masses must be affected. Since the LHS is of the form, $G\,\frac{Mm}{r^2}$ the effect on gravitational mass must be the same as the effect on rulers giving the correct analysis:

$$LHS \qquad \frac{G\,M\,m}{r^2} \rightarrow \frac{1G \times e^{\frac{\Phi_s}{c^2}} M_{ag} \times e^{\frac{\Phi_s}{c^2}} m_{pg}}{\left(e^{\frac{\Phi_s}{c^2}} r\right)^2}$$

So we see that the masses on the left hand side are in fact gravitational masses which are affected in quite a different way by the sun's gravitational potential. Because gravitational potential is additive and its effects are multiplicative, it follows that what is true for the planet moon system is true for the sun planet system.

The orbital radius of the planet moon system is multiplied by a factor $e^{\frac{\Phi_s}{c^2}}$. The orbit shrinks as the planet moon system enters regions where the magnitude of the sun's gravitational potential Φ_s is greater. It is interesting to note that the planet's gravitational potential Φ_p in the region of the moon's orbit is not affected by the changing background gravitational potential through which it moves:

$$\frac{1G \times e^{\frac{\Phi_s}{c^2}} M_{ag}}{e^{\frac{\Phi_s}{c^2}} r} = \frac{G M}{r}$$

We can now apply this to the planet's orbit remembering that Φ_s is a function of the distance from the sun.

- The sun's gravitational mass M_{ag} is affected by its own gravitational potential field and is smaller by a factor of $e^{\frac{\bar{\Phi}_s}{c^2}}$. (Note: we use an average value $\bar{\Phi}_s$ for Φ_s because it varies throughout the body of the sun.)
- The planet's passive gravitational mass m_{pg} is smaller by the factor $e^{\frac{\Phi_s}{c^2}}$.
- The planet's orbit is smaller by the same factor.
- The gravitational equipotential spherical surfaces about the sun are reduced in radius by the same factor.

When we are describing the effects of a gravitational potential field, we say that rulers shrink, so that if we measure the circumference of an orbit with a ruler, the answer will be too big. This simplistic statement does not take into account the fact that an orbit is controlled by physical laws; it is not just a geometrical object. This raises the thorny question which arises in all theories of relativity "What exactly is being mapped onto what?" As the planet moves into regions of different gravitational potential, the moon's orbit contracts and expands. We have to map the orbit onto itself for various values of Φ_s. We cannot map the geometrical circle in Euclidean space onto itself. The planet's orbit is smaller because of the effect of the sun's gravitational potential, but the measured circumference of the orbit has the correct value for use in the formulae:

$$F = \frac{G M m}{r^2} \qquad \Phi = -\frac{G M}{r}$$

The local measurements and calculations give numerical values of physical mass M, m and orbit circumference $2\pi r$ which are all too big by a factor of $e^{-\frac{\Phi_s}{c^2}}$ and these factors cancel to give the correct answer for the force and potential. The correct force and potential may also be calculated from the Euclidean radius and the reduced gravitational mass.

$$\Phi = -\frac{G M_p}{r_c} = -\frac{G M_g}{r}$$

Let us suppose that our sun were attracted by a supermassive "black hole" and started falling towards it. The first effect that we would notice was the slowing of time dependant processes. We would see this first as blue shift in the light from the distant stars. Latter as things became more extreme we would be able to observe the universe for billions of years into its future. If the universe could observe our fate, the effect on time dependent processes would greatly slow our decent into the black hole and we would never be seen to reach it. That would not help our predicament because time for us would be equally slowed so that we would live long enough to see the whole process through its end. Those who believe in Einstein's General Theory of Relativity would have plenty of time to reflect upon the fact that the solar system had passed through the supposed singularity where $1 + \frac{2\Phi}{c^2} = 0$ without any observable effect. Even when the solar system had shrunk to the size of a golf ball, the planets would still be orbiting it and life would be carrying on as normal. Why? How? Because the distance of the solar system from the centre of the supermassive object as calculated from the circumference of its orbit measured with a ruler would still be many many orders of magnitude greater than the size of the solar system, so the variation in gravitational potential and force within the solar system would be negligible.

A full mathematical analysis of the effects of gravitational potential relating to so called black holes is given in the section *Massive neutron stars*.