Massive neutron stars

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The metrics of GR coincide with the much earlier prediction of Rev. Michell that light cannot escape from within a radius $r = \frac{2GM}{c^2}$. GR then predicts and defines an event horizon at this radius. However, the logic behind this depends on the use of weak field approximations and the order in which they are applied giving either $\left(1 - \frac{|\Phi|}{c^2}\right)^2$ or $\left(1 - 2\frac{|\Phi|}{c^2}\right)$. We assert that the expression $\left(1 - 2\frac{|\Phi|}{c^2}\right)$ derived through weak field approximations coincides with the first two terms of the exponential function $e^{2\frac{\Phi}{c^2}}$. This is consistent with our previous proof that the effects of gravitational potential are to multiply by powers of $e^{\frac{\Phi}{c^2}}$.

While Einstein would seem to have worked backwards from Rev. Michell's prediction in formulating his theory, our correction to the metric replacing $\left(1 - 2\frac{|\Phi|}{c^2}\right)$ with $e^{2\frac{\Phi}{c^2}}$ leaves no room for the prediction of an event horizon.

From the above discussion in previous sections, it would seem that there are two factors mitigating against the collapse of a neutron star to a singularity. The first is the nature of the function $e^{2\frac{\Phi}{c^2}}$ and the second is the reduction in gravitational mass. The question is whether or not these are enough to prevent the collapse of a sufficiently massive neutron star. This is further complicated by the fact that as the star collapses, potential energy is converted into kinetic energy. The gravitational mass cannot reduce without a dissipation of the kinetic energy. The collapsing star must therefore radiate energy. It is also reasonable to assume that much of the kinetic energy will be stored in the rotation of the star.

A star does not suddenly become a neutron star, but evolves into one as temperatures and pressures favour electron capture and iron nuclei embark on a decay cycle which turns them into a neutron fluid. If the star is massive enough, a point is reached when the gravitational potential at its centre due to its own mass becomes sufficient to significantly increase the density of physical mass per unit Euclidean volume. However, the effect of gravitational potential in reducing gravitational mass prevents this from turning into a runaway process which might results in a black hole.

As we have seen, gravitational potential is additive and its effects multiplicative. This gives us a method of modelling the process of constructing a neutron star. This is obviously not the way nature constructs neutron stars, but it is a well proven mathematical technique used in the classical calculation of gravitational potential and force. We start with a small core and then build it up by bringing in shells of material from afar. Each new shell of mass $m_i = \delta m$ will have a constant potential inside it equal to $-\frac{G\,\delta m}{r_i}$, so its effect on the assembly of shells within will be to reduce the dimensions of each of the shells which have already been added by a factor $e^{-\frac{G\,\delta m}{c^2 r_i}}$. But the new shell is influenced by the gravitational potential of the existing star which has so far been built up. This means that both δm and r_i are affected and we must calculate the gravitational potential with care. We have shown that gravitational potential can be calculated in two ways:

$$\Phi = -\frac{G\,M_p}{r_c} = -\frac{G\,M_g}{r}$$

either using physical mass M_p and the radius r_c calculated from the circumference as measured with a ruler or gravitational mass M_g and Euclidean radius r .

The first principle of the classical derivation is that the gravitation potential due to a thin spherical shell is constant within that shell giving $\Phi_{i,int} = \frac{G\,m_i}{r_i}$. If we now surround that shell with another shell of mass m_j and radius r_j, we increase the magnitude of the gravitational potential within by $\Phi_{j,int} = -\frac{G\,m_j}{r_j}$ with the result that

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every Euclidean length within m_j is reduced by the factor $e^{\frac{\Phi_j}{c^2}}$. But the gravitational mass of every m_i within m_j is also reduced by the same factor, so every Φ_i within m_j remains constant. That is to say that with the addition of the j_{th} shell, the potential due to the existing i shells, as measured at points within the mass, is unaffected.

Because the locally measured value of the radius r_c of each existing shell remains constant, as additional shells are added we may use the locally measured radius r_c as the independent variable. This allows us to work in terms of the physical mass of the shells. If each new shell has a surface area of $4\pi\, r_c^2$ and a thickness δr_c, its volume will be $4\pi\, r_c^2\, \delta r_c$ in local units of volume. Measuring volume in local units gives a constant numerical result because the unit of length is affected in the same way as is length, and likewise for volume. The physical mass of the shell is therefore:

$$\delta m_p \;=\; 4\pi\, \rho\, r_c^2\, \delta r_c$$

As successive shells are added, a particular shell j is reduced in Euclidean size, however, its numerical thickness, radius, surface area and volume expressed in local units remain constant. The gravitational potential within it, due to its mass also remains constant.

If we were able to wonder around within the star making local measurements with a ruler in order to calculate its volume, we would find everything consistent with the local parameter r_c and would be quite unaware of the distortions of the star's interior relative to Euclidean space. Thus we would find that the volume of the star as the sum of locally measured divisions was equal to $\frac{4}{3}\pi\, R_c^3$ where R_c is the locally measured radius of the star. Our parameter r_c is thus good for calculating the physical mass of the star from its locally measured density ρ (physical mass per local unit volume) and outer radius. This allows us to determine R_c from its physical mass and the density of neutron fluid.

$$R_c \;=\; \sqrt[3]{\frac{3\, M_p}{4\pi\, \rho}}$$

In the classical derivation, the gravitational potentials at the centre and at the surface of a spherical mass of uniform density are:

$$\Phi_c \;=\; -\frac{3\, G\, M_g}{2\, r} \qquad \Phi_s \;=\; -\frac{G\, M_g}{r} \qquad \Phi_c \;=\; \frac{3}{2}\Phi_s$$

and the mathematical derivation of them using the Euclidean measurements and the independent variable r have a one to one correspondence with our calculations in terms of our parameter r_c, so we may conclude that:

$$\Phi_c \;=\; -\frac{3\, G\, M_p}{2\, r_c} \qquad \Phi_s \;=\; -\frac{G\, M_p}{r_c}$$

And we can take the classical result for the gravitational potential at some distance a from the centre of sphere and within its mass and perform the same mapping to give:

$$\Phi_a \;=\; -G\, M_g\, \frac{3\, r^2 \;-\; a^2}{2\, r^3} \;\rightarrow\; \Phi_a \;=\; -G\, M_p\, \frac{3\, r_c^2 \;-\; a_c^2}{2\, r_c^3}$$

Thus we can determine the effects of gravitational potential within the star in terms of these parametric values.

Substituting $R_c \;=\; \sqrt[3]{\frac{3\, M_p}{4\pi\, \rho}}$

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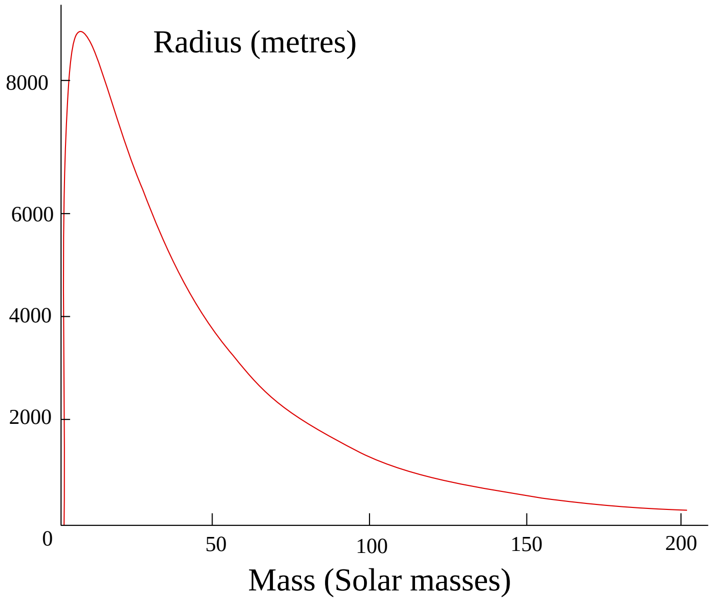
$$\Phi_s = -G M_p \sqrt[3]{\frac{4\pi\rho}{3 M_p}} = -G \sqrt[3]{\frac{4\pi M_p^2 \rho}{3}}$$

The Euclidean radius of the star is thus:

$$R = e^{\frac{-G \sqrt[3]{\frac{4\pi M_p^2 \rho}{3}}}{c^2}} \sqrt[3]{\frac{3 M_p}{4\pi \rho}}$$

This is a well behaved function, however the nature of the exponential function means that as a neutron star increases in mass, a point is reached when its Euclidean radius starts to decrease. Because of the power of the exponential function, very massive stars will collapse towards a very small, but finite size. The maximum Euclidean diameter of a neutron star is just under 19 km for 5.26⊙ (⊙ unit equal to mass of sun is read "solar masses") and one of 883⊙ would have the Euclidean size of a golf ball. The following graph was copied form one generated by Mathcad.



Recent observations of the motion of stars close to the centre of the galaxy indicate the existence of a very massive black hole estimated by UCLA Galactic Centre Group as 3,700,000⊙. On substitution of such a large number into the above formula, even Mathcad simply returns 0. To get a value for the Euclidean radius, we need to employ a little pre-calculator knowledge and "take logs".

$$\log R = \log\left(e^{\frac{-G \sqrt[3]{\frac{4\pi M_p^2 \rho}{3}}}{c^2}} \right) + \log\left(\sqrt[3]{\frac{3 M}{4\pi \rho}} \right)$$

$$= \frac{\frac{-G \sqrt[3]{\frac{4\pi M_p^2 \rho}{3}}}{c^2}}{\ln 10} + \log\left(\sqrt[3]{\frac{3 M}{4\pi \rho}} \right)$$

$$= -1710.3712$$

$$= \overline{1711}.6288$$

$$R = 4.245 \times 10^{-1711}$$

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At this point, only those of us old enough to have been taught to use logs to multiply decimals will understand what is going on. A log consists of a characteristic and a mantissa to the left and right of the decimal point. Antilog tables require a positive mantissa, so -1710.3712 becomes $-1711 + 0.6288$ and is written as $\overline{1711}.6288$ and read "bar 1711 point 6288". We would then look up 0.6288 in the antilog table and get 4.254 giving the final answer as 4.245×10^{-1711}.

This is rather small

So we find that the concepts of black holes and singularities become somewhat blurred. What we have is an object which from the point of view of observation is a point sized object, but it is still vastly bigger than a singularity with zero size. The important thing to understand is that even with an object of 4.245×10^{-1711} metres radius, the laws of physics still apply. They do not break down and if scientific rulers and beam balances could be constructed of neutron fluid is would still be possible to measure the density of the neutron fluid and the circumference of the star in local units and calculate its mass. The mathematical functions are well behaved. It may only have a Euclidean radius the size of nothing, but in local units, it has a radius of over a thousand km. If the figure for the density of neutron fluid can be relied upon, then its locally measured circumference would be about 8,700 km.

The most important factor is the effect on time dependent processes. In particular, the effect on the process of forming a massive neutron star. The mathematics is too complicated to produce a meaningful model, but we can make some rough estimates. If we consider the supermassive object as a sphere of uniform density which is increasing in density as it looses energy. At a stage where it is still 1,000,000 times less dense than neutron fluid, the effect of gravitational potential will be to slow time dependant processes by a factor of about 7×10^{-18}. Now the universe is only about 5×10^{17} seconds old, so since the formation of the earth, less than a tenth of a second of local time has passed. That is only long enough for light to travel half the radius of the object. So there is going to be a limiting factor imposed by the age of the universe. When on average the object was 10,000,000 times less dense, time was only slowed by a factor of 10^{-8} and a second of local time lasted about 4 months, so we can assume that the limiting process cuts in somewhere between these states, say at 2,000,000 times less dense which gives a locally measured radius of about 175,000 km and a Euclidean radius of about 0.004 mm.

Not so small after all!

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