Force on a Charge in a Magnetic Field

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In our unified theory, the primary function of magnetic fields is to provide elementary charged particles with the property of inertial mass. We assert that the electric fields of elementary charged particles coexist in space forming a background against which the motion of an individual charged particle generates a magnetic field $\vec{B} = \mu_0 \vec{u} \wedge \vec{D}$ and that the energy stored in its magnetic field is its kinetic energy. The inertial force is generated by the need to do work, (or have work done to) to change the energy content of the magnetic field. While the inertial forces associated with linear acceleration arise simply from energy considerations, the explanation of centrifugal force resisting centripetal acceleration requires us to postulate that energy moves parallel to the electric field. Thus an element of area of the surface sits at the base of a conic element and a force acts on (or by) the surface element as a result of the changes in magnetic energy density within the conic element.

We wish to use the same methods to prove that the force acting on a charge q moving with velocity $\vec{v}$ through a magnetic field which in the absence of the charge has a flux density $\vec{B}$ is:

$$F = B\,e\,v \qquad \text{or} \qquad \vec{F} = q\,\vec{v} \wedge \vec{B}$$

We assert that the fundamental equation of the electromagnetic interactions is:

$$Q_m = \frac{1}{2}\mu_0 \left(\sum_i \vec{u}_i \wedge \vec{D}_i\right)^2$$

where Q_m is the energy density of the universal magnetic field which results from the motions $\vec{u}_i$ of the electric fields $\vec{D}_i$ of all elementary charged particles. All magnetic forces are due to changes in Q_m requiring work to be done to moving charges, or allowing them to do work.

We can partition this equation: that is to say that we separate the summation into summations over various sets of charges. We choose to separate the moving charge q_j from all the other charges whose motions sum to give the magnetic intensity $\vec{H}_0 = \sum_{i \neq j} \vec{u}_i \wedge \vec{D}_i$ of the magnetic field in the absense of q_j.

$$Q_m = \frac{1}{2}\mu_0 \left(\vec{H}_0 + \vec{u}_j \wedge \vec{D}_j\right)^2 \tag{1}$$

The velocity $\vec{u}_j$ of the charge is measured relative to the background because it is this which generates the magnetic field containing its kinetic energy and it is against this that any force on the charge acts doing work or adsorbing energy.

We can also substitute $\vec{D}_j = \frac{q\,\hat{r}}{4\pi r^2}$. It is now clear that the only charge in question is q_j, so we can drop the subscripts giving:

$$\begin{aligned} Q_m &= \tfrac{1}{2}\mu_0 \left(\vec{H}_0 + \vec{u} \wedge \frac{q\,\hat{r}}{4\pi r^2}\right)^2 \\ &= \tfrac{1}{2}\mu_0 \left(\vec{H}_0^2 + 2\,\vec{H}_0 \cdot \vec{u} \wedge \frac{e\,\hat{r}}{4\pi r^2} + \left(\vec{u} \wedge \frac{q\,\hat{r}}{4\pi r^2}\right)^2\right) \end{aligned}$$

Further substitution gives:

$$Q_m = \tfrac{1}{2}\vec{B}_0 \cdot \vec{H}_0 + \vec{B}_0 \cdot \vec{u} \wedge \frac{q\,\hat{r}}{4\pi r^2} + \tfrac{1}{2}\vec{B}_q \cdot \vec{H}_q$$

Where $\vec{B}_0$ and $\vec{H}_0$ describe the magnetic field through which the charge q is travelling with velocity $\vec{v}$ while $\vec{B}_q$ and $\vec{H}_q$ describe the magnetic field which would surround the moving charge in the absence of $\vec{B}_0$. Thus the first term gives the energy density of the background field in the absence of the moving charge and the third term gives the kinetic energy of the moving charge. The middle term is therefore responsible for any change in the energy density. Differentiating with respect to time:

$$\frac{d}{dt} Q_m = \frac{d}{dt}\left(\vec{B}_0 \cdot \vec{u} \wedge \frac{q\,\hat{r}}{4\pi r^2}\right)$$

We consider the energy content of a conic element described by spherical polar co-ordinates $(r, \theta, \varphi)$with origin at the charge's instantaneous position. We orientate the co-ordisnate system such that $\vec{B}$ lies in the $\varphi = 0$ plane and $\vec{v}$ lies in the $\theta = 0$ axis. The conic element has a solid angle $\delta\omega = sin\,\theta\,\delta\varphi\,\delta\theta$ and a volume element $\delta\tau = r^2\,\delta\omega\,\delta r$. If the energy content of the conic element which is due to the second term is $\delta\mathcal{E}_m$, then The rate of change of energy within the conic element due to the second term is:

$$\frac{d}{dt}\delta\mathcal{E}_m = \int_{r_0}^{\infty} \frac{d}{dt} Q_m\, r^2\, \delta\omega\, dr$$

$$\frac{d}{dt}\delta\mathcal{E}_m = \int_{r_0}^{\infty} \frac{d}{dt}\left(\vec{B}_0 \cdot \vec{u} \wedge \frac{q\,\hat{r}}{4\pi r^2}\right) r^2\, \delta\omega\, dr$$

$$\frac{d}{dt}\delta\mathcal{E}_m = \frac{q}{4\pi}\,\delta\omega \int_{r_0}^{\infty} \frac{d}{dt}\left(\vec{B}_0 \cdot \vec{u} \wedge \hat{r}\right)\, dr$$

Consider the term $\vec{B}_0 \cdot \left(\gamma_u \vec{u} \wedge \hat{r}\right)$. It is a vector triple scalar product which gives a scalar field ϕ in space. This scalar field has the same locus as the magnetic field $\vec{B}_0$. The volume elements are moving through that scalar field at a velocity $\vec{v}$, the velocity of the charge relative to the magnetic field. The rate of change of the scalar field within the volume elements is given by $\vec{v} \cdot \nabla\phi$. The symbols $\nabla\phi$ are called the gradient and are usually read as "Grad phi". The gradient of a scalar field is a vector field and the rate of change depends on the speed and direction, so we take the scalar product $\vec{v} \cdot \nabla\phi$. Then:

$$\vec{v} \cdot \nabla\phi = \vec{v} \cdot \nabla\left(\vec{B}_0 \cdot \vec{u} \wedge \hat{r}\right)$$

It follows that:

$$\frac{d}{dt}\delta\mathcal{E}_m = \frac{q}{4\pi}\,\delta\omega \int_{r_0}^{\infty} \vec{v} \cdot \nabla\left(\vec{B}_0 \cdot \vec{u} \wedge \hat{r}\right)\, dr$$

We now have two distinct velocities for our charge, $\vec{v}$ its velocity relative to the magnetic field through which it is moving and $\vec{u}$ its velocity through the the background against which motion generates magnetic intensity. For Maxwell, this would have been the aether, but in our unified theory, the electric fields of all elementary charged particles coexisting in space form that background. Although $\vec{u}$ is unknowable, it appears later in our calculations as the velocity involved in the generation and adsorption of energy by the surface of the charge. It then becomes a simple matter to eliminate it leaving only the relative velocity $\vec{v}$.

We still have a problem to resolve with our use of co-ordinates. It is necessary to express $\vec{v} \cdot \nabla\phi$ in spherical polar co-ordinates.

$$\vec{v} \cdot \nabla = v_x \frac{d}{dx} + v_y \frac{d}{dy} + v_z \frac{d}{dz}$$

$$= v_x \frac{d}{dr}\frac{\partial r}{\partial x} + v_y \frac{d}{dr}\frac{\partial r}{\partial y} + v_z \frac{d}{dr}\frac{\partial r}{\partial z}$$

$$= v_x \frac{d}{dr}\frac{1}{cos\,\theta} + v_y \frac{d}{dr}\frac{1}{sin\,\theta\; cos\,\varphi} + v_z \frac{d}{dr}\frac{1}{sin\,\theta\; sin\,\varphi}$$

$$= \frac{d}{dr}\left(\frac{v_x}{cos\,\theta} + \frac{v_y}{sin\,\theta\; cos\,\varphi} + \frac{v_z}{sin\,\theta\; sin\,\varphi}\right)$$

On substitution:

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{q}{4\,\pi}\,\delta\omega \int_{r_0}^{\infty} \frac{d}{dr}\left(\frac{v_x}{cos\,\theta} + \frac{v_y}{sin\,\theta\; cos\,\varphi} + \frac{v_z}{sin\,\theta\; sin\,\varphi}\right)\left(\vec{B}_0 \cdot \vec{u} \wedge \hat{r}\right)\, dr$$

The integrations are now trivial since in general $\int \frac{d}{dx}\left(F\,()\right) dx = F\,()$

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{q}{4\,\pi}\,\delta\omega\left[\left(\frac{v_x}{cos\,\theta} + \frac{v_y}{sin\,\theta\; cos\,\varphi} + \frac{v_z}{sin\,\theta\; sin\,\varphi}\right)\left(\vec{B}_0 \cdot \vec{u} \wedge \hat{r}\right)\right]_{r_0}^{\infty}$$

The magnetic flux density $\vec{B}_0$ of the magnetic field in the absence of the moving charge is a function of r to be evaluated at the limits in the usual manor. At infinity, $\vec{B}_0 = 0$, therefore:

$$\frac{d}{dt}\delta\mathscr{E}_m = -\frac{q}{4\,\pi}\,\delta\omega\left(\frac{v_x}{cos\,\theta} + \frac{v_y}{sin\,\theta\; cos\,\varphi} + \frac{v_z}{sin\,\theta\; sin\,\varphi}\right)\left(\vec{B}_0 \cdot \vec{u} \wedge \hat{r}\right)$$

$\vec{B}_0$ has ceased to be a general descriptor of the magnetic field and now gives the magnetic flux density which would exist at the location of the charge if the charge were not there.

It is now desirable to subject the scalar triple product to a cyclic rotation giving:

$$\frac{d}{dt}\delta\mathscr{E}_m = -\frac{q}{4\,\pi}\,\delta\omega\left(\frac{v_x}{cos\,\theta} + \frac{v_y}{sin\,\theta\; cos\,\varphi} + \frac{v_z}{sin\,\theta\; sin\,\varphi}\right)\left(\vec{u} \cdot \hat{r} \wedge \vec{B}_0\right)$$

$$= -\vec{u} \cdot \left\{\frac{q}{4\,\pi}\left(\frac{v_x}{cos\,\theta} + \frac{v_y}{sin\,\theta\; cos\,\varphi} + \frac{v_z}{sin\,\theta\; sin\,\varphi}\right)\left(\hat{r} \wedge \vec{B}_0\right)\right\}\,\delta\omega$$

$$\frac{d}{dt}\delta\mathscr{E}_m = -\frac{q}{4\,\pi}\,\delta\omega\left(\frac{v_x}{cos\,\theta} + \frac{v_y}{sin\,\theta\; cos\,\varphi} + \frac{v_z}{sin\,\theta\; sin\,\varphi}\right)\left(\vec{u} \cdot \hat{r} \wedge \vec{B}_0\right)$$

$$= -\vec{u} \cdot \left\{\frac{q}{4\,\pi}\left(\frac{v_x}{cos\,\theta} + \frac{v_y}{sin\,\theta\; cos\,\varphi} + \frac{v_z}{sin\,\theta\; sin\,\varphi}\right)\left(\hat{r} \wedge \vec{B}_0\right)\right\}\,\delta\omega$$

We can equate this result with the action $\vec{u} \cdot \delta\vec{F}$ of a force $\delta\vec{F}$ moving with velocity $\vec{u}$. Note that both this action and the generation of the magnetic field possessing the kinetic energy of the charge take place against the background formed by the presence of the electric fields of all elementary charged particles. The force on the element of the surface of the charge is:

$$\delta\vec{F} = -\frac{q}{4\pi}\left(\frac{v_x}{\cos\theta} + \frac{v_y}{\sin\theta\cos\varphi} + \frac{v_z}{\sin\theta\sin\varphi}\right)(\vec{B}_0 \wedge \hat{r})\,\delta\omega$$

We have in effect cancelled a dot product $\vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C} \Rightarrow \vec{B} = \vec{C}$. While this is not allowed directly, it is correct that $IF \quad \vec{A}\cdot\vec{B} = \vec{A}\cdot\vec{C} \quad for\ any\,\vec{A} \quad THEN \quad \vec{B} = \vec{C}$. This is the case for $\vec{u}$.

To find the total force, we must integrate over the surface of the charge.

$$\vec{F} = -\frac{q}{4\pi}\int_0^{\pi}\int_0^{2\pi}\left(\frac{v_x}{\cos\theta} + \frac{v_y}{\sin\theta\cos\varphi} + \frac{v_z}{\sin\theta\sin\varphi}\right)(\vec{B}_0 \wedge \hat{r})\,d\varphi\ \sin\theta\,d\theta$$

$$\vec{F} = -\frac{q}{4\pi}\int_0^{\pi}\int_0^{2\pi}\left[\frac{v_x}{\cos\theta} + \frac{v_y}{\sin\theta\cos\varphi} + \frac{v_z}{\sin\theta\sin\varphi}\right]\begin{pmatrix} B_y\sin\theta\sin\varphi - B_z\sin\theta\cos\varphi \\ B_z\cos\theta - B_x\sin\theta\sin\varphi \\ B_x\sin\theta\cos\varphi - B_y\cos\theta \end{pmatrix} d\varphi\ \sin\theta\,d\theta$$

$$= -\frac{q}{4\pi}\int_0^{\pi}\int_0^{2\pi} v_x\begin{pmatrix} B_y\tan\theta\sin\varphi - B_z\tan\theta\cos\varphi \\ B_z - B_x\tan\theta\sin\varphi \\ B_x\tan\theta\cos\varphi - B_y \end{pmatrix} + v_y\begin{pmatrix} B_y\tan\varphi - B_z \\ B_z\cot\theta\sec\varphi - B_x\cot\varphi \\ B_x - B_y\cot\theta\sec\varphi \end{pmatrix} + v_z\begin{pmatrix} B_y - B_z\cot\varphi \\ B_z\cot\theta - B_x \\ B_x\cot\varphi - B_y\cot\theta\,\mathrm{cosec}\,\varphi \end{pmatrix} d\varphi\ \sin\theta\,d\theta$$

Removing terms which integrate over φ or θ to give zero:

$$\vec{F} = -\frac{q}{4\pi}\int_0^{\pi}\int_0^{2\pi} v_x\begin{pmatrix} 0 \\ B_z \\ -B_y \end{pmatrix} + v_y\begin{pmatrix} -B_z \\ 0 \\ B_x \end{pmatrix} + v_z\begin{pmatrix} B_y \\ -B_x \\ 0 \end{pmatrix} d\varphi\,\sin\theta\ d\theta$$

$$= -q\begin{pmatrix} v_z B_y - v_y B_z \\ v_x B_z - v_z B_x \\ v_y B_x - v_x B_y \end{pmatrix}$$

$$= q\begin{pmatrix} v_y B_z - v_z B_y \\ v_z B_x - v_x B_z \\ v_x B_y - v_y B_x \end{pmatrix}$$

$$= q\,\vec{v} \wedge \vec{B}$$

We have derived from our fundamental assertions the force $\vec{F} = q\,\vec{v} \wedge \vec{B}$ upon a moving charge

We do however make the proviso that the charge q is surrounded by its own magnetic field and does not sit in the background magnetic field. The flux density $\vec{B}$ is that in the absence of the charge. An alternative form of the equation is:

$$\vec{F} = q\,\vec{v} \wedge \mu_0\vec{H}_0$$

It should also be emphasised that the force does not result from a direct action of ether $\vec{B}$ or $\vec{H}_0 = \sum_{i\neq j}\vec{u}_i \wedge \vec{D}_i$ upon the charge. These terms arise from the integration of over the whole field.