Ampere's Law

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Ampere's law takes two forms. It was originally formulated as:

$$\oint \vec{H} \cdot d\vec{l} = I$$

In this form it is important in determining the energy content of a quantum strand of magnetic flux. It states that the integral around a closed path of the component of the magnetic intensity $\vec{H}$ which lies in the path is equal to the current I which threads the path. It follows that the energy content of the magnetic field generated by a current is $\mathcal{E} = \frac{1}{2} \Phi I$.

Its other form uses the vector field function of differentiation called *Curl*.

$$Curl\, \vec{H} = \vec{J}$$

It is usually met as one of Maxwell's equations when they are expressed in differential form and equates the *Curl* with the current density $\vec{J}$. The function is easy to calculate, but difficult to imagine because it does not necessarily involve rotation! A curl results where $\vec{H}$ varies in intensity perpendicular to its direction.

In our formulation of our unified theory, we have replaced this one of Maxwell's equations with the more fundamental:

$$\vec{H}_i = \vec{v}_i \wedge \vec{D}_i$$

to take into account the nature of electric current as consisting of moving electrons. (While this form lacks Maxwell's concept of a displacement current needed to derive the wave motion of light, it admits to a more simple interpretation in which the electric and magnetic flux of a photon or radio wave move at the speed of light)

The expression for the action $\vec{H}_i = \vec{v}_i \wedge \vec{D}_i$ of a moving charge involves the velocity $\vec{v}_i$. Velocity is a relative measure and strictly speaking $\vec{v}_i$ should be measured relative to the background formed by the electric fields of all elementary charged particles. However in the study of electromagnetism we are considering currents through wires and for every conduction band electron, there are a set of elementary charged particles which form an associated lattice ion. If we take the actions of their motions into account, the net action is that generated by the velocity of the conduction band electron relative to the conductor.

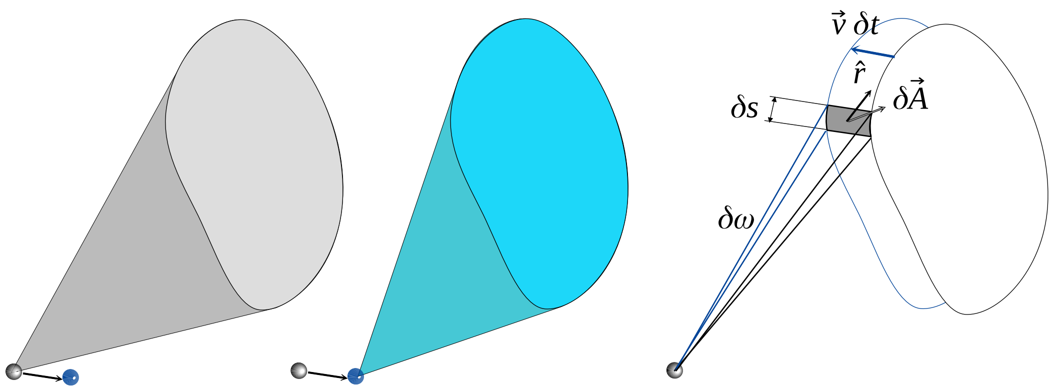
We will now consider a closed path around which we perform the integral $mmf = \oint \vec{H} \cdot d\vec{l}$. This could well be the path of a quantum fluxoid loop which is part of a magnetic field, but it could also be an arbitrary path. If we now consider a single charge q moving in the region of the loop, its motion may be considered to generate a magnetising effect $\vec{H}_i = \frac{q\, \vec{v}_i \wedge \hat{r}_i}{4\pi r_i^2}$ which contributes to the *mmf* around the path of the loop. This contribution may be positive or negative depending on the geometry.

$$mmf_i = \oint \frac{q\, \vec{v}_i \wedge \hat{r}_i}{4\pi r_i^2} \cdot d\vec{s}$$

$$= \frac{q}{4\pi} \oint \frac{d\vec{s} \wedge \vec{v}_i \cdot \hat{r}_i}{r_i^2}$$

The integral $\frac{1}{4\pi}\oint \frac{d\vec{s} \wedge \vec{v}_i \cdot \hat{r}_i}{r_i^2}$ has a simple geometric interpretation and is equal to the rate of change of the solid angle subtended from the charge to the loop. In the diagram below, we show the charge moving through a distance $v\,\delta t$. The solid angles before and after are shown by the grey and blue cones. The blue cone appears shorter (because in the diagram, the charge is moving towards the loop) corresponding to an increase in the solid angle ω. The third picture shows one element $\delta\omega$ of the change $\Delta\omega$ in solid angle. We have shown positions of the loop relative to the charge. The area $\delta\vec{A}$ is that swept by the motion (relative to the charge) of the segment $\delta\vec{s}$ of the loop in time δt and is given by the vector cross product and expressed as a vector perpendicular to the area. The element of solid angle subtended depends on the relative directions of $\delta\vec{A}$ and the radius vector $\hat{r}$ and is given by:

$$\delta\omega = \frac{\delta\vec{A} \cdot \hat{r}}{r^2} \qquad \delta\vec{A} = \delta\vec{s} \wedge \vec{v}\,\delta t$$



By multiplying top and bottom of our expression for the *mmf* by δt we get

$$\frac{1}{\delta t}\frac{d\vec{s} \wedge \vec{v}_i\,\delta t \cdot \hat{r}_i}{r_i^2} = \frac{1}{\delta t}\,\delta\omega$$

This is the change in solid angle due to one element of the loop. The integral around the loop gives the total change in $\Delta\omega$ of the solid angle subtended.

$$\oint \frac{d\vec{s} \wedge \vec{v}_i \cdot \hat{r}_i}{r_i^2} = \frac{1}{\delta t}\,\Delta\omega$$

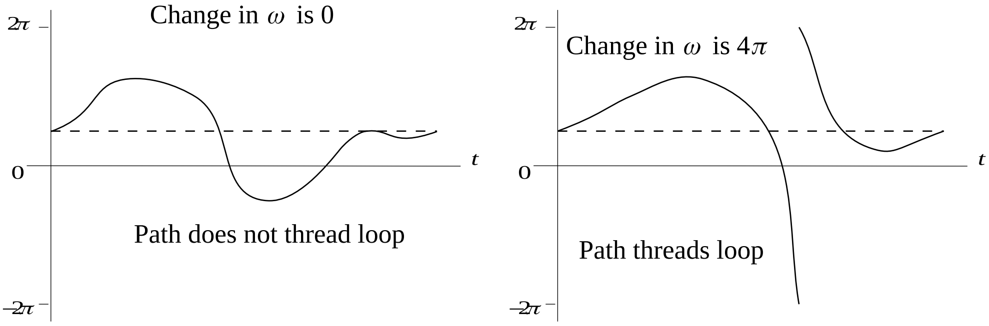
$$mmf_i = \frac{q}{4\pi}\oint \frac{d\vec{s} \wedge \vec{v}_i \cdot \hat{r}_i}{r_i^2} = \frac{q}{4\pi}\frac{d\omega_i}{dt}$$

The subscript i denoting that this is the *mmf* due to the motion of a single electron. We are concerned with a current I flowing around a circuit.

$$mmf = \sum_i mmf_i$$

Each individual conduction band electron is bouncing around in all directions at great speed within the conductor. This makes it impossible to perform the summation at an instant in time. However, over hours or even days, each conduction band electron will complete a complete journey around the circuit. Its average contribution to the *mmf* in this time will be the total change in solid angle subtended divided by the time. If the loop is not threaded by the circuit, then the total change in solid angle is zero.

If however the circuit threads the loop, there is a point as it passes through the loop where the solid angle subtended becomes $\pm 2\pi$ as the solid angle changes from being in front of the electron to being behind it. It changes sign. It would be wrong to think of this as a sudden change of 4π because this is an artefact of the way in which we define the sign of a solid angle. We are concerned with $\frac{d\omega_i}{dt}$ and it is this which changes sign as it passes through the loop. Thus the total change in solid angle subtended is 4π.



The *mmf* due to a current I results from the sum of the average actions of the $\frac{I}{q}$ conduction band electrons which pass through a point on the conductor per second:

$$\text{Therefore} \qquad mmf = \frac{q}{4\pi} 4\pi \frac{I}{q} = I$$

This is Ampere's law:

$$\oint \vec{H} \cdot d\vec{l} = I$$

The alternative form $Curl\ \vec{H} = \vec{J}$ where $\vec{J}$ is a current density comes from the geometric definition of Curl. The curl of a vector $\vec{A}$ is found by taking the integral $\vec{A} \cdot d\vec{s}$ around a small path and dividing by the area within the path. The plane of the path is varied to obtain the maximum value and the limit found as the size of the path tends to zero. Now the current through such a small path is $\vec{J} \cdot \delta\vec{A}$. The maximum value will be obtained when $\vec{J}$ and $\delta\vec{A}$ are parallel, so:

$$Curl\ \vec{H} = \underset{\delta A \to 0}{limit} \frac{\oint \vec{H} \cdot d\vec{l}}{\delta A} = \underset{\delta A \to 0}{limit} \frac{\vec{J} \cdot \delta\vec{A}}{\delta A}$$

$$Curl\ \vec{H} = \vec{J}$$

Both forms of Ampere's law have been derived form our fundamental assertions