Relativity: Fact and Fiction

A causal theory of Special Relativity based on the work of Lorentz and Poincaré

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Abstract

We examine the two theories of Lorentz-Poincaré relativity and Einstein's special relativity. One a causal theory based on Maxwell's Equations, the other pure mathematics derived from a philosophical assumption. The historical roots of special relativity are examined and evidence given to suggest that Einstein's theory was plagiarised from the work of Lorentz Poincaré and others. Based on the ideas of Lorentz and Poincaré, we develop the theory with full mathematical rigor from Maxwell's equations through contraction in length, increase in mass, effect on clock rate and clock synchronisation errors to derive the Lorentz transforms from the stationary to a moving systems. We show that Poincaré's group theory analysis does not yield a group, but then prove that the use of the Lorentz transforms is valid between any two moving systems. We examine the differences between these two theories and highlight the flaws in Einstein's theory. Our understanding of the nature of magnetic fields, the nature of the background and the calculation of kinetic energy are discussed.

History

The theory of relativity was not invented by Einstein. It evolved through the work of a number of men over about fifteen years. Anyone interested in the history should read the two volume edition of Whittaker's 'History Of The Theories Of The Aether And Electricity'[1]. The two leading men were Lorentz and Poincaré. All the elements were in place in early 1905 and available to Einstein when he wrote his 1905 paper. He took Poincaré's relativity principle and produced some neat mathematical fudges to derive the relativity equations from it. Whittaker points out that Einstein's only original contribution was the relativistic Doppler effect[1i].

The theory was developed in response to the failure of experiments to detect the earth's motion though what Maxwell had described as "the luminiferous medium" which he understood to be the seat of the electric and magnetic fields[2i]. Just what the 'luminiferous medium' was remains a mystery whatever name it is given. Maxwell proved that the speed of light depended on the electrical and magnetic properties of the æther (luminiferous medium) called permittivity and permeability determined the speed of light. Some speculated it should be possible to detect the earth's motion through the æther by experiment but both electromagnetic and optical experiments had failed to detect anything. Most notable of these was the Michelson-Morley experiment which needs no further description. Fitzgerald had proposed that the null result could be explained if matter contracted in the direction of motion. The crucial development came with JJ Thompson's discovery of electrons and the identification of beta rays as high speed electrons. Experimental attempts to measure the charge and mass of beta ray electrons showed that they travelled at near light speed and appeared to increase in mass with speed. Lorentz attempted to tie these two factors together in a single theory which predicted the contraction in length and explained the increase in mass. By 1915, more accurate experimental data on the mass increase confirmed Lorentz's theory, but in 1905 the data favoured a rival theory of Abraham[3i].

By its self, Lorentz's theory is about a contraction in length and an increase in mass. Poincaré pointed out that these would result in a slowing of clocks[1iii]. He suggested that clocks could be synchronised by light pulses and showed that this resulted in synchronisation errors. Putting these factors together gave the Lorentz transform equations. These had originally been derived by others[1iv] and shown to preserve Maxwell's equations. It was Poincaré who first speculated that the effects of motion through the æther conspired to make any attempt to detect the motion impossible and described this as the relativity principle[1iii]. The question was how?

The Lorentz transforms were supposed to be valid from the stationary system of the æther to the laboratory. A proper explanation of the null results required the transforms to be universal. In early 1905, Poincaré published a proof based on "Group Theory"[1v]. Later in the year, Einstein published his own much simplified theory[4] based on the assumption that God would want the laws of physics to be the same for all observers. This leads to a very much simplified derivation of the equations of relativity, but it lacks mathematical rigor and its validity is still much debated.

The great mystery is as to why Lorentz acclaimed Einstein's theory and abandoned his own[1ii]. Perhaps he did not understand Poincaré's group theory, perhaps he saw it was flawed, perhaps the incorrect data on the mass increase decided the issue. (Einstein was "cleaver" enough to cover both the results of Lorentz and Abraham by stating that it all depends on the way mass is defined.[4i]) It was also the case that Lorentz's theory of the mass increase was flawed, but that is easily corrected.

The Lorentz contraction

Lorentz identified two of the fundamental equations of electricity and magnetism as being special cases of the same equation[3iii]:

Maxwell's wave equation $$\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varphi = 0$$ and Poisson's equation $$\nabla^2\varphi = \frac{\rho}{\varepsilon_0}$$

are special cases of $$\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varphi = \frac{\rho}{\varepsilon_0}$$

Now $\frac{\partial^2}{\partial t^2} = \frac{dx}{dt}\frac{\partial}{\partial x}\left(\frac{dx}{dt}\frac{\partial}{\partial x}\right) = v^2\frac{\partial^2}{\partial x^2}$. Upon expanding $\nabla^2\varphi$ and collecting terms

$$\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2}{\partial x^2}\varphi + \frac{\partial^2}{\partial y^2}\varphi + \frac{\partial^2}{\partial z^2}\varphi = \frac{\rho}{\varepsilon_0}$$

Making the substitution $x = \sqrt{1 - \frac{v^2}{c^2}}\,x' \qquad y = y' \qquad z = z'$

$$\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial x'^2} \quad : \quad \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial y'^2} \quad : \quad \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial z'^2}$$

reduces the equation to $$\frac{\partial^2}{\partial x'^2}\varphi + \frac{\partial^2}{\partial y'^2}\varphi + \frac{\partial^2}{\partial z'^2}\varphi = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \quad \nabla^2\varphi = \frac{\rho}{\varepsilon_0}\ in\ x'y'z'$$

This is interpreted as meaning that if we have a system, held in equilibrium by electrostatic forces when at rest and described by co-ordinates x, y, and z, that, when the system is in motion, its condition of equilibrium is governed by the same equations written in the new co-ordinates x', y', and z'.

A distance $\delta x'$ measured in the moving system remains the same as it would be when the system was at rest in the stationary system because the ruler we use to measure it has also suffered a contraction. If we had some god-given ruler which was not affected by motion through the stationary system it would measure the distance to be shorter:

$$\partial x = \sqrt{1 - \frac{v^2}{c^2}}\,\partial x'$$

The cause of the contraction in length is a feedback process between the electric and magnetic fields. It is the same feedback process which allows photons and radio waves to exist and travel at the speed of light.

The moving electric field of a photon generates a magnetic field and the moving magnetic field in turn generates the electric field. At the speed of light, the two actions are self sustaining. Electrons have their own electric fields, so they do not need to move at the speed of light in order to exist, but the feedback mechanism is there never the less and affects the electric field intensifying it and changing its shape. The feedback is unable to alter the charge of an electron, so its total electric flux is unaltered. The result is that the surface of the electron and its electric field as described by $\vec{D}$ and ϕ is Lorentz contracted. This is a real contraction caused by a real velocity through the background.

It is customary to use either the symbol β or γ defined as:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Sometimes the factor γ is involved in an increase as in mass; sometimes in a decrease as in length. Since γ is always bigger than 1, it acts either as a multiplier or divisor . When we say something is increased by a factor γ, we mean multiplied by γ. When we say it is decreased by a factor γ, we mean that it is divided by γ.

The mass increase

To explain why mass increases as the speed of light is approached, we first need a theory which explains what mass is. The discovery of electrons had raised hopes that all mater might consist of nothing but electric charges[1]. Lorentz's theory of electromagnetic mass assumes that a moving electric charge generates a magnetic field according to Maxwell's laws and that the energy contained in the magnetic field is the kinetic energy of the charge.

Mass is not so much a "substance" or "essence" which matter is made of, but a property matter appears to posses by virtue of the fact that moving charges possess kinetic energy. To accelerate an atom, we have to apply a force which does work to generate more kinetic energy. To decelerate an atom, we must allow it to exert a force against a resisting force so that it looses kinetic energy doing work against the resistance.

The kinetic energy $\mathcal{E}_m = \frac{1}{2} m v^2$ of an electron is stored in its magnetic field:

$$\vec{B} = \mu_0 \vec{v} \wedge \vec{D} = \frac{\mu_0 q}{4 \pi r^2} \vec{v} \wedge \hat{r}$$

$$\mathcal{E}_m = \int_{volume} \frac{1}{2\mu_0} B^2 = \frac{\mu_0 q^2}{12 \pi a} v^2$$

where a is the radius of the electron.

As we have said, the motion of the magnetic field generates an electric field. In this case, it would be more correct to say it generates an "electric effect" which acts on the existing electric field of the electron causing it to contract in length in the direction motion. The overall result is that the magnetic field is contracted. It still contains the same quantity of magnetic flux, but the contraction increases its flux density B increasing energy density by a factor of γ^2 which is partially cancelled by the decrease in volume. The kinetic energy is increased:

$$\mathcal{E}_m = \frac{\mu_0 q^2}{12 \pi a} \gamma v^2$$

The correct experimental result is that the mass appears to increase by a factor γ when derived from the deflection of an electron by a magnetic field which produces centripetal acceleration, but by a factor of γ^3

when accelerated in its direction of motion by an electric field. Originally, these two apparently different masses were called transverse mass and longitudinal mass writing $m_t = \gamma m_0$ and $m_l = \gamma^3 m_0$, though this has now been dropped in favour of $m = \gamma m_0$, dealing with the increased longitudinal effect by redefining concepts.

The reason for the two apparent masses is that we define mass as the property by which matter resists acceleration. We infer the mass from the acceleration produced by a force. In the case of linear acceleration, the force has to do work to increase the kinetic energy. For small velocities we might write $F = \frac{d}{dt}\frac{1}{2}m v^2$, but for near light speeds, we must use $F = \frac{d}{dt}\frac{1}{2}m \gamma v^2$ and since γ is a function of v we have:

$$F = \frac{1}{2}m \frac{d}{dt}\left(\frac{v^2}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = \gamma^3 m\, a_x$$

where a_x is the acceleration in the direction of motion.

Lorentz's theory was incomplete[5i]. According to electric theory at the time, the contraction in length should also have produced an increase in the energy contained within the electron's electric field. At the time. Poincaré produced a fudge saying that the increase would be balanced by a decrease in the energy stored in the internal structure of the electron. We can now explain that this is not the case. Lorentz's derivation applies to the potential ϕ as a descriptor of the electric field. The electric field had two other descriptors, $\vec{D}$ the electric flux density and $\vec{E}$ the electric field intensity. Now, $\vec{E}$ is a function of the electric potential: $\vec{E} = \nabla\phi$. These are vector quantities: they have properties of direction and magnitude. The energy density of the electric field is $\frac{1}{2}\vec{D} \cdot \vec{E} = \frac{1}{2} D E \cos\theta$ where θ is the angle between $\vec{D}$ and $\vec{E}$. The true effect of the contraction is to rotate $\vec{E}$ towards the line of motion and $\vec{D}$ away from it increasing the angle θ between $\vec{D}$ and $\vec{E}$. While D and E both increase in magnitude by a factor γ, the term $\cos\theta$ and the capacity of the volume element each decreases by the same factor with the net result that the energy in the electric field remains constant.

Poincaré's internal stresses do not exist. Classical electrostatic teaching includes two rival theories of the self energy of the electron, one in which it is contained in the external electric field, the other based on the energy stored in the mutual repulsion of the surface elements of charge. If both were true, the self energy would be double its experimental value. The electron is a single entity with a surface from which an electric field projects outwards. Its surface elements do not exert any force on each other because they do not sit in each other's electric field whereas the electrons on the surface of a metal sphere do sit in each other's fields.

Lorentz's result $m_t = \gamma m_0$ and $m_l = \gamma^3 m_0$ was correct. There is a background, whatever that is, and motion through that background causes a contraction in length through an electromagnetic effect. This in turn increases mass. In his 1906 lectures (later to be published with notes in 1915 as "The Theory of Electrons"[3i]) Lorentz deferred to Abraham's result $m_t = \gamma^2 m_0$ and $m_l = \gamma^4 m_0$ latter adding a footnote to the published lectures to correct this.

The slowing of clocks

We do not say that time is affected. Clocks and measurements of time are affected. This is part of the philosophical difference between Lorentz-Poincaré relativity and Einstein's theory. We assume that there is an ultimate reality which we try to measure. Einstein assumes that it is the observation that is real. An assumption which is no more than thinly disguised existentialism.

We might be tempted to give Einstein the credit for his light clock derivation of the effect on clocks in which they appear to slow by a factor of γ, but as Whittaker refers to the principle being used earlier by Voigt, Fitzgerald, Larmor and Lorentz[1iv] this is to give credit where it is not due. If we introduce the light clock into Lorentz-Poincaré relativity, then the light clock in the moving system really does run slow. When the light pulse travels back and forth perpendicular to the x axis as seen in the moving system, it really moves in a zigzag through the stationary system and the legs of its journey are longer by a factor γ as can be calculated using Pythagoras Theorem.

Clocks are a problem because they are three dimensional machines which suffer a Lorentz contraction in only one dimension!!! So we need three theories as to why a pendulum clock should slow depending whether its pendulum swings perpendicular to the line of motion, parallel to it or if the line of motion is vertical. The historical accident of the incorrect experimental data luckily gave the correct effect on mass for calculating the effect on clocks. An agile mind can work these all out using the contraction in length and the concepts of longitudinal and transverse masses provided they use Abraham's $m_t = \gamma^2 m_0$ and $m_l = \gamma^4 m_0$. Fortunately, pendulum clocks do not work well on ships of either the sea or space going varieties and most clocks use a mechanism involving some form of oscillation executing simple harmonic motion which does not primarily depend on gravity. The formula for SHM requires that the the mass increase by a factor of γ^2 in order to increase the period by a factor γ.

It is interesting to see how SHM varies with direction. When it is parallel to the direction of motion compared to transverse oscillation, the longitudinal mass applies introducing a factor of γ^2. This is countered by two effects of the contraction. The amplitude is reduced by a factor of γ and the gradient of the potential which produces the force is increased by a factor γ.

One of the more fundamental forms of clock is a planet. It orbits and spins with fixed periods and our measurement of time is fundamentally a mapping of events onto the simultaneous orbital and rotational state of the earth. We might imagine a flywheel mounted on frictionless bearing in a vacuum and use its rotations as a clock on our space ship. If we assume it has constant rotational kinetic energy, it too slows as predicted due to the increase of its moment of inertia.

$$\mathcal{E}_{rot} = \tfrac{1}{2} m k^2 \omega^2 \rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m k^2}{2\,\mathcal{E}_{rot}}}$$

where k the radius of gyration and $\mathcal{E}_{rot}$ the rotational kinetic energy are constant. The period T varies as the square root of the mass, so to cause T to increases by a factor of γ, the mass must increase by a factor of γ^2.

Abraham's result fits nicely with our understanding of the relationship between period of oscillation and mass in vibrating systems. We are forced to conclude that within the moving system, mass is apparently increased by a factor of γ^2, but that for one reason or another, which we will explore latter, when we try to observe this from the stationary system, it seems that $m_t = \gamma m_0$ and $m_l = \gamma^3 m_0$.

Looking at the past

These days, we measure the speed of our personal computers in GHz. The clock inside a 1 GHz PC beats once every nanosecond. Light travels just under 1 foot in a nanosecond, so for the sake of illustration, we shall pretend that one foot is the same as a light nanosecond. If something is 10ft away, I see it as it was 10 ns ago. OK: so it takes my brain many millions of nanoseconds to process the signal, but we want to be able to use units which the mind can grasp. We also have digital cameras which contain their own computer and can add a time stamp to each photo. It is not too great a step to imagine we have a digital camera which can take a photo in a fraction of a nanosecond and time stamp it. Or even have a video camera which can take one frame every nanosecond and time stamp it.

If we want to be really accurate about timing events which we photograph, all we have to do is to measure how many feet it is from the camera to the object and subtract that number of nanoseconds from the time recorded. Let's pretend clocks have digital displays showing the time in hours, minutes, seconds, milliseconds, microseconds and nanoseconds. Human eyes can only see down to the hundreds of milliseconds digit, all the others are a blur, but we should be able to photograph the clock and read all the digits. Thus, we can see if two clocks are correctly synchronised by photographing them.

In the universe as described by Lorentz and Poincaré, we have a problem because our location on earth is moving. The earth spins and orbits the sun. The sun has a velocity relative to the stars in this part of our spiral arm of the galaxy. The galaxy rotates and moves through space relative to other local galaxies and the whole universe is expanding. Astronomers can see periodic variation in brightness of binary stars thousands of light years away. De Sitter gave this as a proof that the speed of light is constant[6]. We would retreat from such a firm statement saying that photons travel in fairly straight lines and obey a no overtaking rule. This implies that locally, the speed of light is constant. The only reasonable explanation is that there is a background in which the electromagnetic interactions take place. It is most likely that, in our region of space, this background is stationary relative to the centre of our galaxy and to some degree rotating with it. The point is that the earth is moving through the background.

Our calculations of how long the light takes to reach the camera are wrong because the light travels in the background through which the earth is moving.

Synchronisation of clocks

Not only are clocks slowed by motion through the stationary system, but their synchronisation is affected. Clocks can be accurately synchronised in two ways. One is to physically move one clock next to the other and do it electronically by sending a signal from one to the other down a short wire, then return the synchronised clock to its required location. The other is to send a radio signal from one clock to the other, and allow for the time taken for the radio signal to reach it. Both are affected by motion through the stationary system. As a clock is moved around within the moving system, its velocity through the stationary system varies affecting the rate of the clock and causing synchronisation errors. Alternatively, using radio signals, the actual distance travelled through the stationary system by the radio signal is different from the distance measured within the moving system.

The amazing thing is that both factors give exactly the same result.

Moving a clock

A clock is taken on a journey within the moving system. While is is moving within the moving system, its velocity through the stationary system is changed altering the extent to which it runs slow. The difference in clock rates is:

$$\frac{dt'}{dt} = \sqrt{1 - \frac{v^2}{c^2}} - \sqrt{1 - \frac{(v + w_x)^2 + w_y^2 + w_z^2}{c^2}}$$

There is no exact analysis of this, but we can on the assumption that $w \ll v \ll c$ expand each of the square roots into a series with regard to v and perform the subtraction. Omitting higher powers gives:

$$\frac{dt'}{dt} = \frac{2v\,w_x + w^2}{2c^2} + \ldots = \frac{v\,w_x}{c^2} \qquad \text{for} \qquad w \ll v$$

The loss of time is:

$$\delta t' = \int \frac{v\,w_x}{c^2}\,dt = \frac{v}{c^2}\int w_x\,dt = \frac{v\,x_m}{c^2}$$

So the synchronisation error is $\frac{v\,x_m}{c^2}{}_{slow}$ in moving system units and $\gamma\frac{v\,x_m}{c^2}{}_{slow}$ in stationary system units. Note that since the integration is of the velocity of the clock in the moving system, the result is x_m.

Light pulse synchronisation

If at the moment the two origins are coincident, a light pulse is emitted from the origin of the stationary system and travels to a point (x', y', z') in the moving system in a time t as measured in the stationary system, then it travels a distance given by:

$$d^2 = \left(v\,t + \frac{x'}{\gamma}\right)^2 + y'^2 + z'^2$$

Note that we have to change the length x into stationary system units by dividing by γ. This distance is equal to ct in the stationary system, so we can equate the two to form an equation in t and solve it.

$$c^2t^2 = \left(v\,t + \sqrt{1 - \frac{v^2}{c^2}}\,x'\right)^2 + y'^2 + z'^2$$

This is a standard solution: we expand to get a quadratic in t and solve by the formula

$$t = -\gamma\frac{v\,x'}{c^2} + \gamma\frac{\sqrt{x'^2 + y'^2 + z'^2}}{c}$$

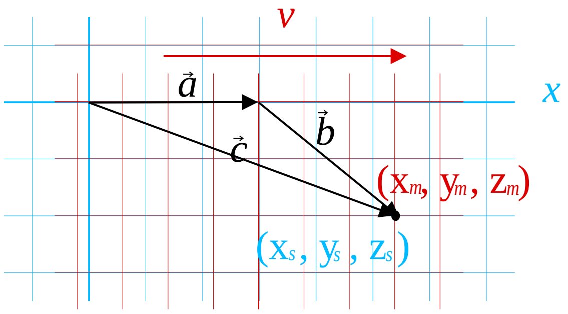
Now $\sqrt{x'^2 + y'^2 + z'^2}$ is just the distance it is seen to travel in the moving system, so dividing by c gives the time t' in moving system units and the factor γ turns the time into stationary system units giving:

$$t = \gamma\left(t' - \frac{v\,x'}{c^2}\right)$$

Thus there is a synchronisation error of $-\gamma\frac{v\,x'}{c^2}$ in stationary system units and $-\frac{v\,x'}{c^2}$ in moving system units. This agrees with the error from moving a clock. It is deliberately written differently to make the reader think about the result.

Transform from stationary to moving system

In relativity, we speak of an event as taking place at a point and at a time. Its descriptor consists of a point and a time recorded in the stationary system as $(x_s, y_s, z_s)@t_s$ or in the moving system as $(x_m, y_m, z_m)@t_m$. The diagram shows the red system moving at a velocity v through the blue stationary system. The x axes are coincident as are the xy planes. This view could have been recorded by a distant camera on the z axis of the stationary system. If we take the event of the light pulse, which was emitted from the origins when they were coincident, arriving at (x_s, y_s, z_s) in the stationary system and (x_m, y_m, z_m) in the moving system.



In the stationary system, the light pulse travels along the vector $\vec{c}$. During this time, the origin of the moving system is seen to travel along the vector $\vec{a}$. In the moving system, the light pulse is seen to travel along the vector $\vec{b}$. Clearly $\vec{a} + \vec{b} = \vec{c}$. This is a vector equation which is true in any co-ordinate system or in any units.

The vector $\vec{b} = \begin{pmatrix} x_m \\ y_m \\ z_m \end{pmatrix}$ moving system units. Vectors $\vec{a} = \begin{pmatrix} v\,t_s \\ 0 \\ 0 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix}$, both in stationary system units. We must convert $\vec{b}$ to stationary system units by dividing its x element by γ before substituting these values in the vector equation:

$$\begin{pmatrix} v\,t_s \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{x_m}{\gamma} \\ y_m \\ z_m \end{pmatrix} = \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix}$$

From this: $x_m = \gamma\left(x_s - v\,t_s\right)$

Within the moving system, the clock at $(x_m,\ y_m,\ z_m)$ is slow compared to the clock at the origin of the moving system by $\frac{v\,x}{c^2}$ moving clock units $(x_m > 0)$, so the moving master clock reads $t_m + \frac{v\,x_m}{c^2}$ which we must convert into stationary system units giving:

$$t_s = \gamma\left(t_m + \frac{v\,x_m}{c^2}\right)$$

We can substitute the above result for x_m and solve for t_m:

$$t_s = \gamma\left(t_m + \frac{v\,\gamma\,(x_s - v\,t_s)}{c^2}\right) \rightarrow t_s = \gamma\,t_m + \frac{\gamma^2\,v\,x_s}{c^2} - \frac{\gamma^2\,v\,t_s}{c^2}$$

$$t_s\left(1 + \frac{\gamma^2 v^2}{c^2}\right) = \gamma\,t_m + \frac{\gamma^2\,v\,x_s}{c^2}$$

But $1 + \frac{\gamma^2 v^2}{c^2} = \gamma^2$ therefore $\gamma^2 t_s = \gamma\,t_m + \frac{\gamma^2\,v\,x_s}{c^2}$

$$t_m = \frac{t_s - \frac{v\,x_s}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Taking this and equating components of the vector equation gives us the Lorentz transform:

$$x_m = \frac{x_s - v\,t_s}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y_m = y_s \qquad z_m = z_s \qquad t_m = \frac{t_s - \frac{v\,x_s}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This derivation describes the physical effects caused by motion though the background in which the electromagnetic interactions take place. We have proved that it works transforming events from the stationary system to the moving system. Lorentz published a derivation of these transforms in 1903 (Proc Amst. Acad)[1vi] based on the invariance of $(c\,t)^2 = x^2 + y^2 + z^2$ and using hyperbolic functions to get the solution.

False symmetry of transform

This is a good point to discuss one of the niggles I have with the Lorentz transforms. The equation for time gives a false impression that time behaves in the same way as length. Particularly in units where $c = 1$ the two equations $x_m = \gamma(x_s - v\, t_s)$ and $t_m = \gamma(t_s - v\, x_s)$ have a high degree of symmetry and swapping x and t changes one into the other. Putting $t_s = 0$ in the first and $x_s = 0$ in the second yields $x_m = \gamma\, x_s$ and $t_m = \gamma\, t_s$. This is most deceptive because while $x_m = \gamma\, x_s$ supports the concept that a moving ruler is contracted in the direction of motion, the second would seem to indicate that a moving clock records a longer time interval. In fact, the reverse is true: a moving clock runs slow and records a shorter time interval.

This causes thinking people a lot of trouble because thinking minds notice logical inconsistencies and work away at trying to understand them. Learning minds just lap up anything they are given and ignore inconsistencies. In fact thinking minds do vast amounts of thinking at the subconscious level popping the results into the conscious mind in eureka moments. Faced with the apparent logical inconsistencies of relativity, the eureka moment never occurs and the conscious mind remains sceptical.

To understand this apparent inconsistency, we need to look back in our derivation to the line:

$$t_s\left(1 + \frac{\gamma^2 v^2}{c^2}\right) = \gamma\, t_m + \frac{\gamma^2\, v\, x_s}{c^2}$$

where the first terms of each side are t_s and $\gamma\, t_m$ and have the correct relationship to each other. The problem is that the effect of synchronisation errors is far greater. So when we take into account the two terms representing the synchronisation error, the left hand side becomes $t_s\left(1 + \frac{\gamma^2 v^2}{c^2}\right)$ which due to the algebra of γ simplifies to $\gamma^2\, t_s$ so that we now compare $\gamma^2\, t_s$ to $\gamma\, t_m$ reversing the relationship.

The reader should ponder this matter for a while: it has taken the author a mere 41 years to come to this eureka moment.

The group theory of Poincaré

Lorentz did not realise that the transform equations would also work from the moving system to the stationary system. The 19th century had been a time of great developments in what we call "Modern Mathematics". In particular, the use of matrices in co-ordinate transformations and the analysis of algebraic structures including group theory. Poincaré[1v] applied this knowledge to the Lorentz transforms and proved to his own satisfaction that together with rotations and translations, the Lorentz transforms formed a group. If this was so then the Lorentz transforms would also be valid from the moving system to the stationary system and between any two moving systems. [This was published on June 5th 1905]

That is to say that given three observers, S in the stationary system and A and B each in a moving system, knowing the transforms from S to A, and from S to B, it should be possible to calculate the transforms between A and B. The problem is that this task seems impossible. The author had earlier disputed the group theory publishing a paper on his web-site and arguing the case in the newsgroup *sci.physics.relativity*. Two years ago, he started work on a more rigorous statement of the case, however much improved functionality of software and hardware together with dogged determination led to a solution of the problem of calculating the transform between A and B. The paper had to change direction.

The author is still of the opinion that the so called "Lorentz Group" is not a group, but never the less, a set of physically meaningful transforms between a number of inertial frames does form a primitive algebraic structure with sufficient properties to ensure that Lorentz transforms are valid between any two inertial frames. This hangs on the fact that the Lorentz transforms are linear transforms which can be expressed through matrix multiplication thus inheriting the algebraic properties of matrix algebra. We shall explore this at length in the following sections.

Lorentz transforms

The Lorentz transforms apply to observers moving in a state of uniform motion. They work when two observers moving relative to one another set up co-ordinate systems according to a set of seven rules:

(i) The points they chose as origin must at some moment be in the same place (coincident).

(ii) They must use the line of sight from one origin to the other as their x axes.

(iii) Their x axes must point in the same direction.

(iv) Their y and z axes must appear parallel as seen looking along the x axes.

(v) They must each have a master clock at their origin.

(vi) They must set their master clocks to zero when their origins are coincident. (This requires a little imagination because in reality, the clocks would collide.)

(vii) They each use light pulses or radio signals between clocks to calibrate and synchronise local clocks spread around their co-ordinate grids.

The Lorentz transform takes into account the effects of motion on rulers and clocks transforming both the xyz position co-ordinates and the time of an event as recorded by a local clock.

If the co-ordinates in the stationary system are x, y, z and t and those of the moving system are x', y', z' and t'. The transform equations are:

$$x' = \frac{x - v\,t}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad y' = y \qquad z' = z \qquad t' = \frac{t - v\,x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The Lorentz transforms are linear transforms since the co-ordinate and time variables appear to the power of 1 in the equations. Linear transforms can be performed by matrix algebra and we might alternatively write:

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\,v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma v & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \qquad : \ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Combining Lorentz transforms

If we have three observers moving relative to one another and they choose origins such that all three are coincident at some moment, then it should be possible for them to set up co-ordinate grids and populate them with clocks according to the rules.

In what might be called a Lorentz-Poincaré universe, we can identify one of the observers S as being at rest relative to the æther. His clocks are all in absolute synchronisation and he can judge a moving line to be parallel to a fixed line because the two lines are coincident at some moment in time. The other two observers A and B will not have their local clocks absolutely synchronised and will see two lines which S judges parallel to be at an angle to each other with the point of intersection moving along the lines as one apparently moves past the other.

The educated reader will have been taught to believe that we live in an Einstein universe where all three systems have equal status and such concepts as stationary and moving are only relative terms. This universe has the property that the laws of physics are the same for all observers leading to the inference that the physical effects of contraction in length, increase in mass and slowing of clocks are not physical effects at all, but are merely artefacts of observation caused by the relative motion of the observers and observed.

If one admits the validity of the Lorentz transforms, then the proof given here will also work in an Einstein universe, but the author disputes the validity of the Lorentz transforms in an Einstein universe. Since in an Einstein universe, no system suffers a real contraction in length, their grid lines beat out perfectly synchronised Newtonian time as they pass one another. From this clocks can be perfectly calibrated and synchronised and the whole basic of Einstein's derivation collapses.

In either case, three observers can set up a total of six co-ordinate grids populated with clocks. We shall refer to a co-ordinate grid populated with synchronised clocks a co-ordinate system. We introduce the notion:

Ab	is a co-ordinate system set up by Observer A aligned with observer B
Ab_P	is an event at point P described in terms of Ab
Cd_Ab	is the transform matrix to change the co-ordinates and time of an event from Ab to Cd

Using this notation, the rules of matrix multiplication allow a cancelling rule.

$$Cd\_Ab \cdot Ab\_P = Cd\_P$$

We can write equations showing both cancelling and expansion using this notation for both a Lorentz transform of an event and the multiplication of one transform matrix by another.

	Transform of point	Multiplication of transforms
Cancelling	$As\_Ab \cdot Ab\_P = As\_P$	$As\_Ab \cdot Ab\_Ba = As\_Ba$
Expansion	$As\_P = As\_Ab \cdot Ab\_P$	$As\_Ba = As\_Ab \cdot Ab\_Ba$

The multiplication of one transform matrix by another is best understood when we apply the transforms to an event.

$$As\_Ab \cdot Ab\_Ba \cdot Ba\_P = As\_Ba \cdot Ba\_P$$

Which is to say that transforming the event P from Ba to Ab and then from Ab to As is the same as transforming it from Ba to As.

Our three observers, S, A and B set up a total of six co-ordinate systems: Sa, As, Sb, Bs, Ab and Ba. There are a total of 36 possible transformations between them: *Sa_Sa*, *Sa_As*, *Sa_Sb*....... *Ba_Ba* which we can divide into various forms:

Six matrices of the form *Ab_As* are rotations.

Six matrices of the form *Ab_Ab* are equal to the identity element I.

The two matrices *Sa_As* and *Sb_Bs* are known to be Lorentz transforms

Another four matrices of the form *Ab_Ba* might be Lorentz transforms.

The remaining transforms are said to be Lorentz invariant. However, they are not proper Lorentz transforms because they do not satisfy rules ii, iii and iv for setting up co-ordinate systems for a standard Lorentz transform, though they do preserve the geometry. Expressed as matrices, they lack the symmetry of Lorentz transform matrices.

The properties of a group are satisfied if we have:

(i) A set

(ii) An operation defined to combine any two members of the set to get another member of the set

(iii) The operation is Associative

(iv) There is an identity element I

(v) Every element has an inverse with which it combines to give I

We must make two points here. The first is that a group is a mathematical entity and its properties are independent of the nature of its elements. The second is that there are a number of ways of defining a group. Mathematical entities often have more properties than are needed to define them and can be defined by different subsets of their properties.

One of the properties of a group is *closure* as implied in (ii). The counting numbers 0, 1, 2, 3.... under addition form a group, but it is an infinite group because if we learn how to count up to a thousand, we might want to add 1000 + 1000 which requires us to extend our number system to count up to 2000. On the other hand, we can get quite small sets which form groups under particular operations. For instance, in the arithmetic of complex numbers, the cube roots of 1 under multiplication form a group with only three members.

Obviously, our set of 36 meaningful transforms and the operation of combining two of them cannot form a group because there is a restriction on combining them. Only operations of the form *As_Ab* · *Ab_Ba* which allow cancellation can be allowed. Therefore there is no closure and the 36 transforms do not form a group.

Our quest is to discover whether or not the two transforms Ab_Ba and Ba_Ab are Lorentz transforms.

The problem has to be set up carefully. Consider three observers S, A and B who can each identify an origin such that the three origins are coincident at some moment and together set up the six co-ordinate systems Sa, As; Ab, Ba; Bs and Sb according to the rules for Lorentz transforms. We ask them to define the positive direction of x to be from S to A; from B to A and from S to B. We ask them to orientate their y axes in the plane of the triangle SAB. Each observer has two co-ordinate systems. We ask them to measure the angles through which they would rotate one x axis to sit on the other.

We can form the two Lorentz transforms Sa_As and Sb_Bs (which we know are valid) and form the rotation matrices which each observer uses between his two co-ordinate systems. Then we can try to form the two transforms Ab_Ba and Ba_Ab by combining transforms. Then we test to see if these transforms are indeed Lorentz transforms and if one is the inverse of the other.

Fig.1 assumes a Lorentz-Poincaré universe and is the view as S, the observer in the stationary system, sees it. Units could be any units in which the speed of light is 1. We are thinking in terms of nanoseconds and light-nanoseconds (slightly under 1 foot) and the view is taken 10 nanoseconds after the origins of the three observer's co-ordinate systems were at O. A stationary system camera at (0, 0, 1000) taking a picture at time

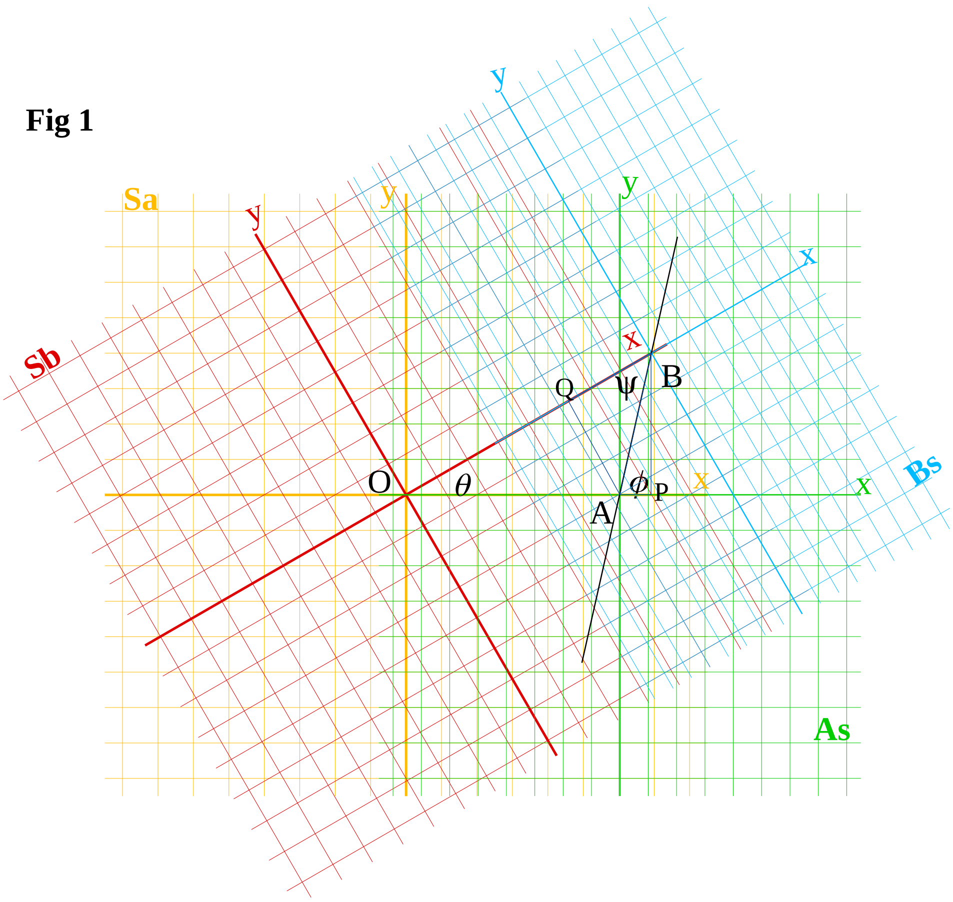
1010ns would record this view.

Only four of the six co-ordinate systems are represented by grids. Sa in yellow and Sb in red are uncontracted. The grid of As in green is contracted by a factor of 4/5 due to its observed speed of 3/5c and that of Bs in blue by a factor of 3/5 due to B's observed velocity of 4/5c.

S observes: speed of A u = 3/5, speed of B v = 4/5 and the angle between their paths to be $\theta = \frac{\pi}{6}$ or 30°. The xy planes of all co-ordinates grids lie in the plane of the triangle OAB.

The clocks of A and B are are affected by Lorentz contraction and mass increase causing them to run slow, so at the time of 10 nanoseconds (By S's master clock) the master clocks of A and B read 8 and 6 nanoseconds respectively. All of S's clocks are in perfect synchronisation and (in the absence of gravity) keep perfect time.

Fig 1



We can write the matrix transforms for As_Sa, Bs_Sb and Sb_Sa:

$$As\_Sa = \begin{pmatrix} \gamma_a & 0 & 0 & -\gamma_a u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma_a u & 0 & 0 & \gamma_a \end{pmatrix} \quad Bs\_Sb = \begin{pmatrix} \gamma_b & 0 & 0 & -\gamma_b v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma_b v & 0 & 0 & \gamma_b \end{pmatrix} \quad Sb\_Sa = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

There is no doubt that we can use matrix algebra to form the transforms Ba_Ab and Ab_Ba and the they will be inverse of each other. The question is whether or not they will be Lorentz transforms. Let us expand the transform *Ba_Ab*:

$$Ba\_Ab \ = \ Ba\_Bs \cdot Bs\_Sb \cdot Sb\_Sa \cdot Sa\_As \cdot As\_Ab$$

The only problem is that we do not know how to measure the angles which have to be used in the rotations Ba_Bs and As_Ab. So we just call them ψ and ϕ, form the matrices and do the multiplication. With a recent edition of Mathcad, that should be simple.

$$\begin{pmatrix} \cos(Bs\_\psi) & \sin(Bs\_\psi) & 0 & 0 \\ -\sin(Bs\_\psi) & \cos(Bs\_\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \gamma_b & 0 & 0 & -\gamma_b \cdot v \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma_b \cdot v & 0 & 0 & \gamma_b \end{pmatrix} \cdot \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \gamma_a & 0 & 0 & \gamma_a \cdot u \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma_a \cdot u & 0 & 0 & \gamma_a \end{pmatrix} \cdot \begin{pmatrix} \cos(As\_\phi) & -\sin(As\_\phi) & 0 & 0 \\ \sin(As\_\phi) & \cos(As\_\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We need three lines to show the result; column 1, column 2 and columns 3 and 4.

$$\begin{pmatrix} \cos(As\_\phi) \cdot \gamma_a \cdot \cos(Bs\_\psi) \cdot \gamma_b \cdot \cos(\theta) - \cos(As\_\phi) \cdot \gamma_a \cdot \sin(Bs\_\psi) \cdot \sin(\theta) - \cos(As\_\phi) \cdot \cos(Bs\_\psi) \cdot \gamma_b \cdot v \cdot \gamma_a \cdot u + \sin(As\_\phi) \cdot \cos(Bs\_\psi) \cdot \gamma_b \cdot \sin(\theta) + \sin(As\_\phi) \cdot \sin(Bs\_\psi) \cdot \cos(\theta) \\ -\cos(As\_\phi) \cdot \gamma_a \cdot \sin(Bs\_\psi) \cdot \gamma_b \cdot \cos(\theta) - \cos(As\_\phi) \cdot \gamma_a \cdot \cos(Bs\_\psi) \cdot \sin(\theta) + \cos(As\_\phi) \cdot \sin(Bs\_\psi) \cdot \gamma_b \cdot v \cdot \gamma_a \cdot u - \sin(As\_\phi) \cdot \sin(Bs\_\psi) \cdot \gamma_b \cdot \sin(\theta) + \sin(As\_\phi) \cdot \cos(Bs\_\psi) \cdot \cos(\theta) \\ 0 \\ -\cos(As\_\phi) \cdot \gamma_b \cdot v \cdot \cos(\theta) \cdot \gamma_a + \cos(As\_\phi) \cdot \gamma_b \cdot \gamma_a \cdot u - \gamma_b \cdot v \cdot \sin(\theta) \cdot \sin(As\_\phi) \end{pmatrix}$$

$$\begin{matrix} -\sin(As\_\phi) \cdot \gamma_a \cdot \cos(Bs\_\psi) \cdot \gamma_b \cdot \cos(\theta) + \sin(As\_\phi) \cdot \gamma_a \cdot \sin(Bs\_\psi) \cdot \sin(\theta) + \sin(As\_\phi) \cdot \cos(Bs\_\psi) \cdot \gamma_b \cdot v \cdot \gamma_a \cdot u + \cos(As\_\phi) \cdot \cos(Bs\_\psi) \cdot \gamma_b \cdot \sin(\theta) + \cos(As\_\phi) \cdot \sin(Bs\_\psi) \cdot \cos(\theta) \\ \sin(As\_\phi) \cdot \gamma_a \cdot \sin(Bs\_\psi) \cdot \gamma_b \cdot \cos(\theta) + \sin(As\_\phi) \cdot \gamma_a \cdot \cos(Bs\_\psi) \cdot \sin(\theta) - \sin(As\_\phi) \cdot \sin(Bs\_\psi) \cdot \gamma_b \cdot v \cdot \gamma_a \cdot u - \cos(As\_\phi) \cdot \sin(Bs\_\psi) \cdot \gamma_b \cdot \sin(\theta) + \cos(As\_\phi) \cdot \cos(Bs\_\psi) \cdot \cos(\theta) \\ 0 \\ \sin(As\_\phi) \cdot \gamma_b \cdot v \cdot \cos(\theta) \cdot \gamma_a - \sin(As\_\phi) \cdot \gamma_b \cdot \gamma_a \cdot u - \gamma_b \cdot v \cdot \sin(\theta) \cdot \cos(As\_\phi) \end{matrix}$$

$$\begin{matrix} 0 & \gamma_a \cdot u \cdot \cos(Bs\_\psi) \cdot \gamma_b \cdot \cos(\theta) - \gamma_a \cdot u \cdot \sin(Bs\_\psi) \cdot \sin(\theta) - \cos(Bs\_\psi) \cdot \gamma_b \cdot v \cdot \gamma_a \\ 0 & -\gamma_a \cdot u \cdot \sin(Bs\_\psi) \cdot \gamma_b \cdot \cos(\theta) - \gamma_a \cdot u \cdot \cos(Bs\_\psi) \cdot \sin(\theta) + \sin(Bs\_\psi) \cdot \gamma_b \cdot v \cdot \gamma_a \\ 1 & 0 \\ 0 & -\gamma_b \cdot v \cdot \cos(\theta) \cdot \gamma_a \cdot u + \gamma_b \cdot \gamma_a \end{matrix} \Bigg)$$

All we have to do is to see if we can equate this to a standard Lorentz transform matrix and solve for ψ and ϕ.

[There are two problems: fear and lack of computing capacity. As stated above, the author's early attempts to solve this problem failed due to lack of computing capacity and software functionality. Returning to the problem in March 2005, the author eventually cracked the problem.]

If this is a Lorentz transform, then the opposite corner elements should be equal and the rest equate to 0 or 1. Equating elements row 2 col 4 and row 3 col 3 to zero gives:

$$\tan\phi \ = \ \frac{v \sin\theta}{\gamma_a (v \cos\theta \ - \ u)} \qquad \tan\psi \ = \ \frac{u \sin\theta}{\gamma_b (u \cos\theta \ - \ v)}$$

Dividing row 4 col 1 by row 4 col 4 gives the speed of A and B which they measure relative to each other

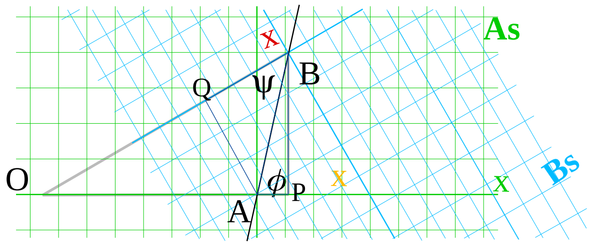
$$w \ = \ \frac{\sqrt{(1 \ - \ u\, v \cos\theta)^2 \ - \ (1 \ - \ u^2)(1 \ - \ v^2)}}{1 \ - \ u\, v \cos\theta}$$

It is not possible to use Mathcad to substitute these values back into the matrix and reduce it to a Lorentz transform matrix because the algebra is too complex. It is however a simple matter to obtain numerical solutions to particular examples and substitute them. This been done enough times to ensure the solution is correct. For the example given in the diagram:

$$\psi \ = \ 32.70° \qquad \phi \ = \ 73.82° \qquad w \ = \ 0.5702\, c \qquad \gamma \ = \ 1.217$$

In hindsight, we can see that these angles could have been obtained from fig.1. They are not the angles which we would measure from fig 1 with a protractor, but are the angles A and B would measure with their contracted protractors. The contracted grids preserve the tangents of the angles.

Fig 2



The enlarged part of the diagram (Fig.2) shows two right angled triangles APB and BQA with the blue and green grids. From this, it is easy to derive the equations:

$$\tan\phi = \frac{PB}{AP} = \frac{v\,t\sin\theta}{\gamma_a(v\,t\cos\theta - u\,t)} \qquad \tan\psi = \frac{QA}{BQ} = \frac{u\,t\sin\theta}{\gamma_b(u\,t\cos\theta - v\,t)}$$

The constants $\gamma_a = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ and $\gamma_b = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$ respectively turning stationary system units into green and blue grid units.

The angles θ, $(\pi - \phi)$ and ψ of the triangle OAB as measured by S, A and B do not add to 180° because A and B measure their angles with protractors contracted in the direction of their motion through the stationary system.

Thus we have proved that:

The validity of the Lorentz transforms from the stationary system to a moving system guarantees that they are valid between any two moving systems.

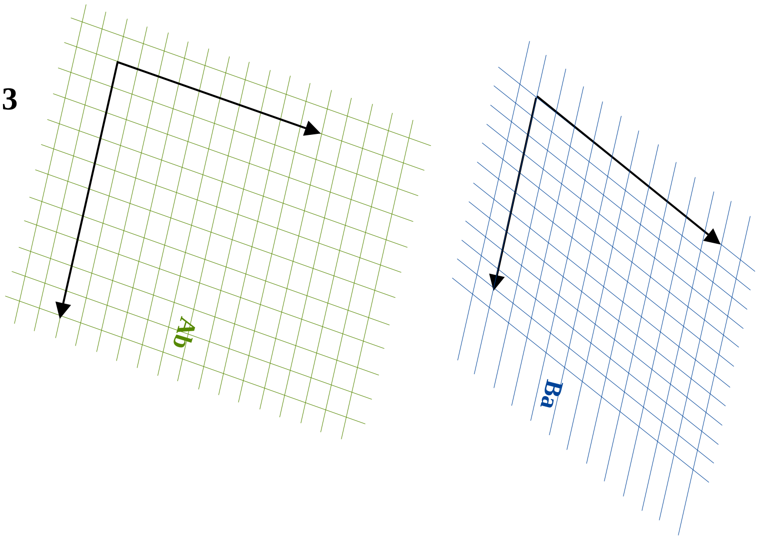
Interpretation

Real observers can only pass each other at small fractions of the speed of the light. To reach a speed of $\frac{4}{5}c$, a starship would need to start off with $\frac{1}{5}$ of it's mass as antimatter and another $\frac{1}{5}$ of its mass as disposable mass to combine with the antimatter. For a their and back journey, it needs four such stages. Even with a fusion reactor which could turn hydrogen into iron a starship would only be able to reach $\frac{1}{100}c$. That puts the distortions caused by the the Lorentz contraction into perspective. The speeds we have used here to produce nice graphics are completely unrealistic, and have only been used to make the effects easy to see.

The next diagram shows the grids Ab and Ba as they would be seen by a camera at rest in the stationary system some distance away on the z axis. Note that they are distorted. We have drawn in vectors of $10\hat{\imath}$ and $10\hat{\jmath}$. The $10\hat{\imath}$ vectors are parallel but not of the same length. The two $10\hat{\jmath}$ vectors are of different length and direction. The question is how do A and B both observe each other's $10\hat{\jmath}$ vectors to be equal and how do they both observe each other's $10\hat{\imath}$ vector to be be contracted by a factor of 0.8215.

The answer is that these diagrams do not show the clock synchronisation errors. They are the view as seen from the stationary system. Observers A and B do not see these views of each other. Each has his own set of synchronisation errors. We have to refer back to Fig.1 to see the light green grid of As and light blue grid of Bs. The synchronisation errors are proportional to the x co-ordinates of these grids.

Fig 3



Observers A and B each see the other's vectors move past their own. But each vector has two ends and the clock synchronisation error is different at each end. The observers see the vector smeared through time according to their clock synchronisation errors. It does not make any difference whether they use very local cameras to record the passing of individual points, or take a long range video from far out on their z axis and examine it frame by frame, they will see the same thing. We can take this into account by using the Lorentz transforms between A and B. If we take the moment when the origins are coincident and form a two column matrix containing B's $10\hat{i}$ and $10\hat{j}$ vectors at time zero and multiply by *Ab_Ba*:

$$\begin{pmatrix} 1.217 & 0 & 0 & 0.694 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.694 & 0 & 0 & 1.217 \end{pmatrix} \begin{pmatrix} 10 & 0 \\ 0 & 10 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 12.17 & 0 \\ 0 & 10 \\ 0 & 0 \\ 6.941 & 0 \end{pmatrix}$$

We find that A will see the point of B's $10\hat{j}$ to be at (0, 10, 0) at time zero, but will see the point of his $10\hat{j}$ vector at (12.17, 0) at time 6.941. In that time, the foot of this vector will have travelled at a speed which A measures as 0.5702 for 6.941 seconds to (3.958, 0, 0), so the length of the vector as seen by A is the difference between these equal to 8.215. We can perform similar calculations for B's view of A's vectors.

Our view of the universe is distorted by the finite speed of light. The further away things are, the further in the past they were when the light we see left them. Most of the scenes we view are local enough for the time light to takes to reach us to vary by only a few microseconds at most which is imperceptible, so we think we see things as they happen. A scenario where the effects of motion and distance smear the image through time are conceptually alien and make these matters hard to understand.

Fig.3 shows the pictures of grids Ab and Ba as seen by a stationary system camera at (0,0,1000). The camera would see them superimposed with their x axes coincident. A camera belonging to A at a similar position on his z axis would show the dark green grid undistorted and the dark blue grid contracted in the x direction. Similarly, a camera belonging to B would show the dark blue grid undistorted and the dark green grid contracted in the x direction. These two camera views automatically add in the synchronisation errors, each consequently seeing their own version of the result of the two motions through the stationary system.

Einstein's errors

Einstein's 1905 paper on special relativity[4] is most probably plagiarised from the work of Lorentz and Poincaré. Because it contains their results in the form of mathematical equations, it is hard to criticise within the framework of the scientific method which demands that theory produce equations which stand the test of experimental verification. However:

(i) Einstein is wrong to say that the speed of light is a universal constant.

(ii) Einstein is wrong to say "The introduction of a 'luminiferous ether' will prove to be superfluous inasmuch as the view here to be developed will not require an 'absolute stationary space' provided with special properties,"

(iii) Einstein is wrong to say that space and time are distorted.

The speed of light is a very interesting phenomena. It does vary, but that variation is impossible to measure in any local experiment. There is a non local experiment in which the speed of light is measured via the exchange of radio signals between earth and a space probe on the other side of the sun. While the earth and the probe follow well defined orbits, the radio signals are found to be delayed when they pass close to the sun. The speed of light measured over the scale of planetary orbits shows experimental variation. Locally along its path, it would always be measured to be the same numerical value because the rulers and clocks used to measure it would be affected by gravitational potential in the same way.

If we could measure the local one way speed of light, we would find it added to our speed through the stationary system, but we cannot do this because we have no way of synchronising two clocks to time the one way speed of light between two points. As we have seen, even placing two clocks side by side, synchronising them and then moving them apart causes synchronisation errors. These errors will always conspire to give the same numerical result for a one way measurement of the speed of light. Two way measurements to a mirror and back will always be affected by the length contraction and the slowing of clocks to give the same numerical result. Einstein's error is to assume that this numerical result is the speed of light. It is a measurement of the speed of light. Thus it is legitimate to say that 'The "locally measured speed of light" is a universal constant.' It is an error to précis that statement to 'The speed of light is a constant.'

The key to understanding Einstein's theory is the ownership of light. All his derivations require one system to be called stationary and the other moving. The trick is to make the stationary system own the light. The observer in the moving system then uses stationary system light to synchronise his clocks. By this trick Einstein temporarily gives to his two systems the properties of the stationary and moving systems of a Lorentz-Poincaré world. Properties which in his interpretation of "the relativity principle" may not exist. (Einstein most probably plagiarised the relativity principle from Poincaré who published[1ii] it in "The Principle of Relativity" Bull. des Sc. Math xxviii 1904) His derivations are a fudge because he uses this ownership of light trick to justify using $c + v$ and $c - v$ in his equations in spite of the fact that he latter asserts that the result of such sums must always be c.

Einstein's basic assertion is that their is no "privileged system" by which he means that there is no æther which can have a physical effect on bodies moving through it. Therefore according to Einstein's axioms; clocks cannot be slowed, rulers cannot contract and mass cannot increase. These according to Einstein are artefacts of observation caused by observing objects in relative motion, not real physical effects. As such they appear to be reciprocal. Both observes see the other's clocks running slow. Both observers see the other's rulers to have contracted in the direction of motion. Both observers find moving objects harder to accelerate and say their mass is increased. Einstein gives no reason for this other than the will of God. No privileged system, therefore no physical effect. For Einstein, it is simple: God created a universe in which the laws of physics would be the same for all observers.

In the real Lorentz-Poincaré world, the effects of motion through the background and the way we see things and synchronise our clocks conspire as we have seen, such that both observes see the other's clocks running slow, their rulers to have contracted in the direction of motion and find moving objects harder to accelerate inferring that their mass is increased.

The result of Einstein's subversion of Lorentz Poincaré relativity is to turn logical causal events into paradoxes. In the real Lorentz-Poincaré world, the twin who takes a trip on a starship really does come back to find his twin has aged more. In Einstein's imaginary universe in which there can be no physical effect, an ingenious fudge is needed. In latter years with the introduction of the General Theory of Relativity, the "slowing of time" is attributed to the acceleration. Mathematically it cannot go wrong as an explanation because integrating acceleration gives velocity and it is velocity through the background of the real universe which causes time dependent processes to slow. Physically, the use of acceleration is nonsense because there are three types of acceleration: linear acceleration, linear deceleration and centripetal acceleration. The acceleration fudge depends on matter knowing which type of acceleration is taking place, yet Einstein not only fails to understand that there are three types of acceleration, but states that acceleration and gravity are indistinguishable.

All the arguments used by Einstein and those who teach his theory hinge on the problems of clock synchronisation. In the real Lorentz-Poincaré universe, clock synchronisation errors occur because of the real physical effects we have discussed, but in an Einstein world, there can be no physical effects, only artefacts of observation. In Einstein's illustration of a train moving down a track, the train does not physically contract. Therefore if two trains pass each other, the passing of their carriage ends will beat out perfectly synchronised Newtonian time allowing local clocks to be calibrated and synchronised. With both trains populated with perfectly synchronised clocks all running at the same speed, no relativistic effects should be observed.

The fact that we observe the relativistic effects proves we live in a Lorentz-Poincaré world.

Magnetism

Einstein's paper of 1905 was called The Electrodynamics Of Moving Bodies. His derivation of the Lorentz transforms is only a small part of the paper. He starts with a criticism of the laws of electromagnetism in which he complains that two theories are required to describe the interaction between a magnet and a circuit, one theory is used when a wire is moved past a magnet, but a different theory is required when the magnet is moved past the wire. It must be said that modern teaching in engineering departments does not make this distinction. Maxwell, however regarded the magnetic field as being stationary in the æther, so that in the case of a moving magnet, there is a continuous process of the rear of the field decaying as the front grows in strength.

Einstein's solution to this problem was drastic. He denied the existence of the magnetic field as a physical entity and reduced it to the rank of an artefact of observation[4iii]. For an engineer building power station generators, this would have been an obvious nonsense, but in the rarefied atmosphere of a university mathematics department pursuing theoretical physics with a sense of distrust in experimental physics, it was not such a bad idea. We might take as an example the explanation of why a wire carrying an electric current might exert a force on a similar parallel wire. This is to be found in standard texts and is regularly taught at university.

The electric current in the wire consists of moving electrons. It is claimed that because they are in motion, the whole set appears contracted in the direction of motion. The electrons are consequently closer together and the wire is said to carry a net negative charge. The positive lattice ions in the other wire are supposedly attracted by this net negative charge and hence the wires are attracted towards each other. That is what is taught. It is of course nonsense. The drift speed of electrons in a wire carrying an electric current is measured in millimetres per hour, not exactly close to the speed of light! The extra electrons would have to come from somewhere: perhaps heaven! We might alternatively argue that the electric fields of the individual moving electrons are contracted and so more intense, but this encounters two problems, first that the average field would be unaffected because the total electric flux would remain constant, secondly even if the averaging process did not take place, it is based on the false assumption that D and E are basically the same thing. The electric flux density D is not responsible for the electric force. The force results from the change in electric potential with change of position. The nature of the contraction does not affect the component of $\vec{E} = \nabla\phi$ perpendicular to the current.

In the SI system of units, the descriptors D and E of the electric field have different dimensions as do B and H of the magnetic field. Magnetic fields are generated as a result of the motion of elementary charged particles through the background. $\vec{H} = \sum_i \vec{v}_i \wedge \vec{D}_i$, the sum being taken over all elementary charged particles. It is wrong to think of $\vec{H}$ as a physical entity. The physical entities are the moving electric fields of the elementary charged particles. $\vec{H}$ is just a mathematical artefact describing the sum of the effects. In response to $\vec{H}$ a magnetic field of flux density $\vec{B} = \mu_0 \vec{H}$ is formed. The magnetic flux is a physical entity and contrary to Maxwell's understanding, its locus is that of the electric circuit or ferromagnetic material which generates it. In taking the summation $\sum_i \vec{v}_i \wedge \vec{D}_i$ the elementary charged particles can be divided into sets whose net contribution is zero. For those in the electric circuit, conduction band electrons can be paired with lattice particles and the absolute velocities then subtract to give the velocity of the conduction band electrons relative to the circuit or magnet. The same applies to a magnet, pairing orbital electrons and lattice ions.

A sophisticated electromagnetic theory can be deduced on the basis that each electron contributes to the energy density of the magnetic field according to its contribution $\vec{H}_i = \vec{v}_i \wedge \vec{D}_i$ to the generation of the magnetic field with energy flowing within its electric field parallel to $\vec{D}_i$. This analysis leads to a rigorous derivation of the laws of induction in which they are seen to be a consequence of the nature of inertia.

The irony is that Maxwell is shown to be wrong. Magnetic flux does not have its seat in the æther, it is a physical entity in its own right with definite locus and velocity through the stationary system. That is not to say that Maxwell's equations are wrong: the mathematical analysis yields the same equations whether we consider the magnetic flux to be stationary or moving. There are two ways of looking at a wave, one is to look at the water level at a point and see it going up and down, the other is to see the wave moving over the water surface. Whichever view one might prefer, the mathematics of wave motion remains the same.

What is the stationary system

The author advanced a theory around 1995/6 that the electric fields of all elementary charged particles coexisted in space. The fact that the magnetic action of a current is described by $\vec{H} = \sum_i \vec{v}_i \wedge \vec{D}_i$ is an obvious proof of the fact that electric fields move through each other and therefore coexist in space.

Despite rigorous attempts to ignore them, experimental evidence is growing to suggest that the solar system does have a definite velocity through space. This takes two forms, the asymmetry of the background blackbody radiation and the timing of pulsar signals. Any local variations in the movement of the background such as those discussed in the days of debate about æther theory would have shown up in the pulsar measurements. If we were to contribute to the late 19th Century debate about the æther, we probably would err in favour of a dragged æther theory in which its motion is influenced by galaxies.

Coupling this with the observations showing that rulers, clocks and light are affected by gravitational potential, the author is now of the opinion that electric potential is the property of the electric field which determines the locus of the background. We can thus define the velocity of the background as:

$$\vec{s} = \frac{\sum \vec{v}_i \, |\phi_i|}{\sum |\phi_i|}$$

The nature of this summation being that the $\vec{v}_i$ may be measured in any convenient frame and the summation taken over all elementary charged particles. The result is a weighted average velocity. In practice we can replace electric potential with gravitational potential. Because of the $\frac{1}{r}$ dependence of potential, the sum is insensitive to local variation and the mass of the galactic core dominates.

We know the galaxy is spinning and this will obviously affect the summation such that we can say that the background to some extent rotates with the galaxy. In theory, a big enough laser gyro would be able to detect zero rotation relative to the background and compare this with the distant galaxies. Our analysis of inertial mass suggests that centrifugal force is proportional to rotation relative to the background. Astronomic observations indicate that centrifugal force should tear the spiral arms away from the galaxy. Astronomers have tried to explain the stable state of the galaxy with dark matter theories. A much simpler explanation is that in the region of the spiral arms, the background rotates with the galaxy, but at a slower angular velocity.

Kinetic energy

It is our assertion that every elementary charged particle has an absolute velocity through the background and that it has an absolute kinetic energy stored in the magnetic field generated by its motion through the background. In the branch of applied mathematics called mechanics, we calculate kinetic energy using velocities measured relative to some arbitrary frame of reference. We find that the law of conservation of energy applies. How can that be true if our calculations of kinetic energy $E = \frac{1}{2} m v^2$ uses arbitrary velocities. The answer is simple. The kinetic energy of a system of particles is equal to the kinetic energy of the system as whole plus the kinetic energy of the particles within the system. If the particles of the system have mass and velocity denoted by m_i and $\vec{v}_i$ measured relative to the centre of gravity of the system which has a velocity $\vec{u}$ through the background, then:

$$\sum_i \tfrac{1}{2} m_i (\vec{u} + \vec{v}_i)^2 = \tfrac{1}{2}\left(\sum_i m_i\right) u^2 + \sum_i m_i \vec{u} \cdot \vec{v}_i + \sum_i \tfrac{1}{2} m_i v_i^2$$

$$\text{Now } \sum_i m_i \vec{u} \cdot \vec{v}_i = \vec{u} \cdot \sum_i m_i \vec{v}_i = 0$$

$$\therefore \sum_i \tfrac{1}{2} m_i (\vec{u} + \vec{v}_i)^2 = \tfrac{1}{2}\left(\sum_i m_i\right) u^2 + \sum_i \tfrac{1}{2} m_i v_i^2$$

The necessary condition for this to be so is that $\sum_i m_i \vec{v}_i = 0$. That is to say that the sum of the momentums of the particles measured relative to the centre of gravity of the system is zero. Since that is how the centre of gravity is defined, the condition is met. The principle of conservation of momentum as expressed by the equation $\sum_i m_i \vec{v}_i = 0$ is a direct consequence of conservation of energy.

This principle works in exactly the same way if we have multiple systems embedded within each other, so just as we might identify the velocity of an electron as its velocity within the atom plus the velocity of the atom relative to the earth plus the velocity of the earth around the sun plus........ there are corresponding systems of system of systems....... to which this principle can be applied.

In relating this to the laboratory situation where we equate $force \times distance$ to a change in kinetic energy, we are measuring only that part of the kinetic energy due to motion within the within the laboratory. We are also only measuring distance within the laboratory. In reality the distance the laboratory has travelled through the stationary system in that time needs to be added to give the real change in kinetic energy. The other end of the force is anchored in the laboratory and dose (or adsorbs) an equal, but opposite signed, amount of work changing the real kinetic energy of the earth. This is an interesting concept: forces on moving objects do work. Once we accept the idea of a background and absolute local velocity, then all forces are doing work. Fortunately, forces usually come in equal and opposite pairs so no net amount of work is done. It is only in situations where one of a pair of equal and opposite forces is an inertial force that we need to worry about the situation. Resolving the universe into systems and applying the law for calculating the kinetic energy, our worries are resolved.

Appendix

Mathematics of the mass increase

[Much of this section has been copied from previous papers, but the mathematics includes some neater solutions and is set out in full.]

We can prove that the energy content of the electric field of the electron is invariant using Cartesian Co-ordinates. The contraction increases the y and z components of the electric flux density $\vec{D}$ and increases the x component of the electric field strength $\vec{E}$ which is equal to the gradient of the potential $\nabla\phi$. The volume element $d\tau$ is also decreased.

$$\vec{D}' = \begin{pmatrix} D_x \\ \gamma\, D_y \\ \gamma\, D_z \end{pmatrix} \qquad \vec{E}' = \nabla\phi' = \begin{pmatrix} \gamma\, \frac{d\phi}{dx} \\ \frac{d\phi}{dy} \\ \frac{d\phi}{dz} \end{pmatrix} \qquad d\tau' = \frac{1}{\gamma}\, d\tau$$

And $$\mathscr{E}_e' = \int \frac{1}{2}\, \vec{D}' \cdot \vec{E}'\, d\tau' = \int \frac{1}{2} \begin{pmatrix} D_x \\ \gamma\, D_y \\ \gamma\, D_z \end{pmatrix} \cdot \begin{pmatrix} \gamma\, \frac{d\phi}{dx} \\ \frac{d\phi}{dy} \\ \frac{d\phi}{dz} \end{pmatrix} \frac{1}{\gamma}\, d\tau = \int \frac{1}{2}\, \vec{D} \cdot \vec{E}\, d\tau$$

Thus the energy content of the electric field of the electron is invariant.

Lorentz performed his integrations using the auxiliary co-ordinates. The raw result were $m_l = \gamma^4 m_0$ and $m_t = \gamma^2 m_0$. He then argued that as the real co-ordinates were contracted, he needed to divide by a factor of γ to give the final result $m_l = \gamma^3 m_0$ and $m_t = \gamma\, m_0$. This led him to reject his own theory in favour of Abraham's theory. For our purposes here, we would prefer not to have to divide by γ leaving the result as $m_l = \gamma^4 m_0$ and $m_t = \gamma^2 m_0$ because this is consistent with the effect on time dependent processes.

The author is of the opinion that in relating the auxiliary co-ordinate solution to the moving system, if we take into account the fact that volume elements are bigger by a factor γ, we should also take into account the fact that we have differentiated with respect to time and that the units of time are longer by the same factor. Consequently the rate of change of energy in the calculation is unaffected.

We diverge considerably from Lorentz's simple derivation which does not give an adequate explanation of centrifugal force. Lorentz deduced the relationship between transverse and longitudinal mass. He derived the longitudinal mass from an integration of the total energy content of the magnetic field. This is very much simpler than the approach we have followed. In order to unmask the process by which centrifugal force is generated, we have to consider the rate of change in energy content of the magnetic field. Now there is no net change, only a rotation of the magnetic field as the direction of motion changes (the flux being orthogonal to the direction of motion). The only way to obtain the correct result is to assume that changes in the magnetic field must be accommodated by the movement of energy parallel to the electric field. Nature does not allow the electric field to rotate, so we may construct a volume element outwards from a surface element parallel to $\vec{D}$ and consider the energy content of the magnetic field within it. This gives us a rate of change of energy within the volume element which we equate to a force on the surface element. The centrifugal force exists as the sum of these forces over the surface elements.

Before we can proceed with the derivation, we need to quote two identities of which the second one is not obvious and from the algebraic perspective, it very unusual:

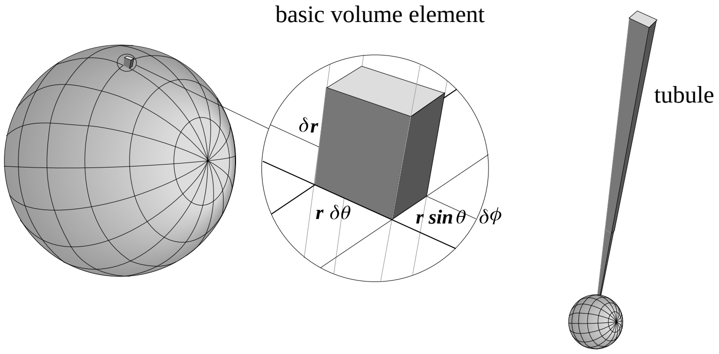
$$\frac{d}{dt}\gamma^n = \frac{n\,v}{c^2}\gamma^{n+2}\frac{dv}{dt} \qquad \text{and} \qquad \frac{v^2}{c^2}\gamma^2 + 1 = \gamma^2$$

We will also need to understand that for functions containing scalars and vectors, the normal rules for differentiating compound functions apply so long as the type and order of multiplication are preserved. Finally we use a technique in which the quadruple scalar product $\vec{A} \wedge \vec{B} \cdot \vec{C} \wedge \vec{D}$ may be treated as a triple scalar product and subjected to cyclic rotation, for example:

$$\vec{A} \wedge \vec{B} \cdot \vec{C} \wedge \vec{D} = \vec{A} \wedge \vec{B} \cdot \left(\vec{C} \wedge \vec{D}\right) = \vec{B} \wedge \left(\vec{C} \wedge \vec{D}\right) \cdot \vec{A}$$

The following derivation is simple when one knows how to do it, but the fact that all of the above need to be used make its discovery almost impossible.

According to classical theory, the moving charge is surrounded by a magnetic field $\vec{B} = \frac{\mu_0\, q}{4\,\pi\, r^2}\vec{v} \wedge \hat{r}$. The Lorentz contraction increases the electric flux density $\vec{D} = \frac{q}{4\,\pi\, r^2}\hat{r}$ by a factor of $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ increasing the magnetic flux density to $\vec{B} = \frac{\gamma\,\mu_0\, q}{4\,\pi\, r^2}\vec{v} \wedge \hat{r}$. The magnetic field has an energy density $\frac{1}{2\,\mu_0}B^2 = \frac{\gamma^2\,\mu_0\, q^2}{32\,\pi^2\, r^4}(\vec{v} \wedge \hat{r})^2$. We consider a surface element of area $\delta A = r^2\,\delta\omega = r^2 \sin\theta\, \delta\theta\, \delta\phi$ and the conical volume element which can be constructed outwards from the surface element everywhere parallel to the electric field. Such volume elements will herein after be referred to as "tubules" since they are constructed according to the rule devised by Faraday to define what latter became known as "Faraday tubes".



We work in auxiliary co-ordinates taking into account the increased flux densities. The basic volume element of the tubule is $\delta\tau = r^2\,\delta\omega\,\delta r$ and the energy content of the magnetic field within the tubule is:

$$\delta\mathscr{E}_m = \frac{\gamma^2\,\mu_0\, q^2}{32\,\pi^2}(\vec{v} \wedge \hat{r})^2 \int_{r_0}^{\infty}\frac{1}{r^4}\, r^2\delta\omega\, dr = \frac{\gamma^2\,\mu_0\, q^2}{32\,\pi^2\, r_0}(\vec{v} \wedge \hat{r})^2\,\delta\omega$$

We can now differentiate this with respect to time to find the rate of change of energy content of the magnetic field within the tubule. We must remember that γ is a function of velocity. Everything which remains constant with time can be left outside the differentiation.

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{d}{dt}\left(\frac{\gamma^2\,\mu_0\, q^2}{32\,\pi^2\, r_0}(\vec{v} \wedge \hat{r})^2\,\delta\omega\right) = \frac{\mu_0\, q^2}{32\,\pi^2\, r_0}\frac{d}{dt}\left(\gamma^2(\vec{v} \wedge \hat{r})^2\right)\delta\omega$$

$$\frac{d}{dt}\gamma^2(\vec{v} \wedge \hat{r})^2 = \frac{d}{dt}\left(\gamma^2\right)(\vec{v} \wedge \hat{r})^2 + \gamma^2\frac{d}{dt}\left((\vec{v} \wedge \hat{r})^2\right)$$

$$= \frac{2v}{c^2}\gamma^4\frac{dv}{dt}(\vec{v} \wedge \hat{r})^2 + 2\gamma^2(\vec{v} \wedge \hat{r}) \cdot \left(\frac{d\vec{v}}{dt} \wedge \hat{r}\right)$$

We write $\frac{d\vec{v}}{dt} = \vec{a}$ as $a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$ and note that at this instant $\hat{v} = \hat{i}$ and $\vec{v} = v\,\hat{i}$. Then $\frac{dv}{dt} = a_x$ since it measures the rate of change in magnitude in $\vec{v}$.

$$\frac{d}{dt}\gamma^2\left(\vec{v} \wedge \hat{r}\right)^2 = 2\gamma^2\left(a_x\frac{v}{c^2}\gamma^2\vec{v} \wedge \hat{r} + \left(a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}\right) \wedge \hat{r}\right) \cdot \left(\vec{v} \wedge \hat{r}\right)$$

The next step is to separate the magnitude v and direction $\hat{i}$ of $\vec{v}$ and rearrange $a_x\frac{v}{c^2}\gamma^2\vec{v} \rightarrow \frac{v^2}{c^2}\gamma^2 a_x\hat{i}$. Then collecting the two terms containing a_x and using the γ^2 identity:

$$\frac{d}{dt}\gamma^2\left(\vec{v} \wedge \hat{r}\right)^2 = 2\gamma^2\left(\left(\gamma^2 a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}\right) \wedge \hat{r}\right) \cdot \left(\vec{v} \wedge \hat{r}\right)$$

Writing $\vec{a}_\gamma = \gamma^2 a_x\,\hat{i} + a_y\,\hat{j} + a_z\,\hat{k}$ and turning the quadruple scalar product into triple scalar product:

$$\frac{d}{dt}\gamma^2\left(\vec{v} \wedge \hat{r}\right)^2 = 2\gamma^2\;\vec{v} \wedge \hat{r} \cdot \left(\vec{a}_\gamma \wedge \hat{r}\right)$$

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{d}{dt}\left(\frac{\gamma^2\mu_0\,q^2}{32\,\pi^2\,r_0}\left(\vec{v} \wedge \hat{r}\right)^2\,\delta\omega\right) = \frac{\mu_0\,q^2}{32\,\pi^2\,r_0}\;2\gamma^2\,\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right) \cdot \vec{v}\,\delta\omega$$

We are now in a position to equate the change in energy with the rate of work done by a force moving with velocity $\vec{v}$ in time δt.

$$\delta\vec{F} \cdot \vec{v}\delta t = \frac{d}{dt}\delta\mathscr{E}_m\,\delta t$$

$$\delta\vec{F} \cdot \vec{v} = \frac{\mu_0\,q^2}{32\,\pi^2\,r_0}\;2\gamma^2\delta\omega\;\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right) \cdot \vec{v}$$

Note that although we chose to impose co-ordinates with the x axis along the direction of motion, this equation requires only that the origin is at the centre of the sphere. Although the dot product does not in general cancel, the fact that this is true for all $\vec{v}$ in this equation implies that:

$$\delta\vec{F} = \frac{\mu_0\,q^2}{16\,\pi^2\,r_0}\;\gamma^2\;\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right)\,\delta\omega$$

Let us remind ourselves that we have just found the force on the surface element of solid angle $\delta\omega$. We may now integrate over the area of the sphere to find the force required to produce the centripetal acceleration.

$$\vec{F} = \frac{\mu_0\,q^2}{16\,\pi^2\,r_0}\gamma^2\int\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right)\,d\omega$$

$$\vec{F} = \frac{\mu_0\,q^2}{16\,\pi^2\,r_0}\,\gamma^2\int_0^{2\pi}\int_0^{\pi}\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right)\,sin\,\theta\;d\theta\;d\phi$$

This calculation is best done in Cartesian co-ordinates expanding the vector product, then integrating. The essentials of have been captured from a Mathcad file.

$$\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\cdot\cos(\phi)\\ \sin(\theta)\cdot\sin(\phi)\end{pmatrix}\times\left[\begin{pmatrix}\gamma^2 a_x\\ a_y\\ a_z\end{pmatrix}\times\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\cdot\cos(\phi)\\ \sin(\theta)\cdot\sin(\phi)\end{pmatrix}\right]=\begin{pmatrix}-\sin(\theta)\cdot\cos(\phi)\cdot a_y\cdot\cos(\theta)-\sin(\theta)\cdot\sin(\phi)\cdot a_z\cdot\cos(\theta)+\gamma^2\cdot a_x-\gamma^2\cdot a_x\cdot\cos(\theta)^2\\ a_y-a_y\cdot\cos(\phi)^2+a_y\cdot\cos(\theta)^2\cdot\cos(\phi)^2-\sin(\phi)\cdot a_z\cdot\cos(\phi)+\sin(\phi)\cdot a_z\cdot\cos(\phi)\cdot\cos(\theta)^2-\cos(\theta)\cdot\gamma^2\cdot a_x\cdot\sin(\theta)\cdot\cos(\phi)\\ a_z\cdot\cos(\theta)^2-\cos(\theta)\cdot\gamma^2\cdot a_x\cdot\sin(\theta)\cdot\sin(\phi)-\cos(\phi)\cdot a_y\cdot\sin(\phi)+\cos(\phi)\cdot a_y\cdot\sin(\phi)\cdot\cos(\theta)^2+\cos(\phi)^2\cdot a_z-\cos(\phi)^2\cdot a_z\cdot\cos(\theta)^2\end{pmatrix}$$

$$\int_0^{2\pi}\int_0^{\pi}\left(-\sin(\theta)\cdot\cos(\phi)\cdot a_y\cdot\cos(\theta)-\sin(\theta)\cdot\sin(\phi)\cdot a_z\cdot\cos(\theta)+\gamma^2\cdot a_x-\gamma^2\cdot a_x\cdot\cos(\theta)^2\right)\cdot\sin(\theta)\,d\theta\,d\phi\rightarrow\frac{8}{3}\cdot\pi\cdot\gamma^2\cdot a_x$$

$$\int_0^{2\pi}\int_0^{\pi}\left(a_y-a_y\cdot\cos(\phi)^2+a_y\cdot\cos(\theta)^2\cdot\cos(\phi)^2-\sin(\phi)\cdot a_z\cdot\cos(\phi)+\sin(\phi)\cdot a_z\cdot\cos(\phi)\cdot\cos(\theta)^2-\cos(\theta)\cdot\gamma^2\cdot a_x\cdot\sin(\theta)\cdot\cos(\phi)\right)\cdot\sin(\theta)\,d\theta\,d\phi\rightarrow\frac{8}{3}\cdot a_y\cdot\pi$$

$$\int_0^{2\pi}\int_0^{\pi}\left(a_z\cdot\cos(\theta)^2-\cos(\theta)\cdot\gamma^2\cdot a_x\cdot\sin(\theta)\cdot\sin(\phi)-\cos(\phi)\cdot a_y\cdot\sin(\phi)+\cos(\phi)\cdot a_y\cdot\sin(\phi)\cdot\cos(\theta)^2+\cos(\phi)^2\cdot a_z-\cos(\phi)^2\cdot a_z\cdot\cos(\theta)^2\right)\cdot\sin(\theta)\,d\theta\,d\phi\rightarrow\frac{8}{3}\cdot a_z\cdot\pi$$

$$\int_0^{2\pi}\int_0^{\pi}\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\cdot\cos(\phi)\\ \sin(\theta)\cdot\sin(\phi)\end{pmatrix}\times\left[\begin{pmatrix}\gamma^2 a_x\\ a_y\\ a_z\end{pmatrix}\times\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\cdot\cos(\phi)\\ \sin(\theta)\cdot\sin(\phi)\end{pmatrix}\right]\cdot\sin(\theta)\,d\theta\,d\phi=\begin{pmatrix}\frac{8}{3}\cdot\pi\cdot\gamma^2\cdot a_x\\ \frac{8}{3}\pi\cdot a_y\\ \frac{8}{3}\pi\cdot a_z\end{pmatrix}$$

[This may be viewed at up to 400% in Acrobat and will print legibly]

Writing the result of the integration as $\frac{8\pi}{3}\vec{a}_\gamma$, the force required to accelerate the electron is:

$$\vec{F} = \frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\gamma^2\,\frac{8\pi}{3}\,\vec{a}_\gamma = \frac{\mu_0\, q^2}{6\,\pi\, r_0}\gamma^2\,\vec{a}_\gamma$$

To relate this solution in the auxiliary co-ordinates to the stationary system, we need to divide by γ to take into account the reduced size of the volume element. Defining a quantity $m_0 = \frac{\mu_0\, q^2}{6\,\pi\, r_0}$ we have the relativistic form of Newton's second law:

$$\vec{F} = \gamma\, m_0\, \vec{a}_\gamma$$

This is the relativistic form of Newton's second law.

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