Inertial Mass as an Electromagnetic Phenomenon

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Abstract

The Lorentz model of the electron is re-examined in classical terms. The theory is advanced: that the electric fields of individual elementary charged particles have separate existence and coexist in space. It is argued that the presence of the individual fields forms a background against which the electromagnetic interactions take place. The kinetic energy of an electron is shown to reside in its surrounding magnetic field and the inertial force resisting acceleration is shown to result from induction as the magnetic field changes. The problem of the infinite flux Φ surrounding the electron is overcome by considering the "substance" of the magnetic field to be energy. It is then possible to calculate the velocity with which magnetic flux emerges from or is adsorbed into the surface of the electron and use Faraday's law to calculate the force exerted by the induced electric field. The Lorentz contraction is shown to be real and to affect the surface of the electron and its electric field as described by $\vec{D}$ and ϕ from which the results $m_t = \gamma\, m_0$ and $m_l = \gamma^3\, m_0$ are derived. The factor $\frac{4}{3}$ in the expression for the inertial mass in terms of the energy content of the electric field is shown to be consistent with $E = m\, c^2$ for photons.

1: Introduction

This paper is the first of a set in which I seek provide a unified theory of action at a distance, electric and magnetic forces, inertia and gravity. Just as the embryo Lorentz-Poincaré relativity based on classical physics was struggling to emerge, Einstein justified its mathematics from a completely different conceptual basis. One of the most amazing incidents in the history of physics was the way in which Lorentz welcomed Einstein's theory of relativity. He could well have denounced him for plagiarism and declared him a fool for abandoning a theory based on real causal effects for one dealing with nothing but artefacts of observation. We can only conclude that the inadequacies of the Abraham-Lorentz models of the electron and the difficulties of reconciling Maxwell's equations with an increasingly discredited aether theory both played a significant role in shaping his response.

Experiments with high energy charged particles presents classical physics with the almost impossible task of explaining why their inertial mass behaved as $m_l = \gamma^3\, m_0$ and $m_t = \gamma\, m_0$. Lorentz nearly achieved this equating the kinetic energy of a spherical charge moving through the aether with the energy content of its magnetic field. His solution was deficient in that the Lorentz contraction of the charge also increased the energy content of its electric field. We will see that Lorentz misinterpreted the nature of the contraction and that the energy content of the electric field remains constant. Modern thinking based on Einstein's ideas rejects the Maxwellian concept of a magnetic field and denies the existence of any background against which the velocity of the charge might result in a quantifiable kinetic energy.

The principle of superposition in classical physics is usually understood as a mathematical theorem allowing us to mathematically treat the electric fields of individual as if they had separate existence while adhering to the doctrine that only one field exists. This way of thinking is rooted in aether theory rather than founded on experimental observation and logical deduction. Although Maxwell's equations have survived the demise of aether theory, their reinterpretation in terms of an aether-less universe has been incomplete. Without an aether, the Poynting vectors become little more than vaguely interesting mathematical artefacts and the validity of retarded potentials becomes very questionable, yet we cling to results derived from them. Feynman's QED may seem to solve the problem of action at a distance, but vast amounts of information have to be stored and processed to insure that a single electron exerts the correct forces.

In this paper, I will develop a classical theory based on an minimalist model of the electron in which it consists of nothing but an electric field in the form of a polarisation of space terminating in an inner surface of displacement charge. In this theory, the electric fields of elementary charged particles all have a separate existence and are extended entities which act locally on charges. In a later paper, I shall show how this model leads to an explanation of the force of gravity. The presence of the myriad of electrical fields in space forms a background against which motion causes electromagnetic interactions. Using this conceptual basis for the nature of charges and their fields, these classical theories predict the same effects on clocks as do SR and GR, on measuring rods as does SR, and the path of photons as does GR.

The conclusion of this paper is at variance with the established results of the electrodynamics based upon the special theory of relativity. It would be premature to dismiss it for this reason alone and it would be just as wrong to use it as an argument against the predictions of SR. The author asks the reader to try to evaluate it from the perspective of early 1905 before Einstein published his paper on relativity.

2: Definitions

SI units are used throughout. $\vec{D}$ and $\vec{E}$ are the microscopic parameters describing fields in vacuum. While being related to each other by the constant ε_0, $\vec{D}$ and $\vec{E}$ are physically very different parameters. $\vec{D}$ describing the state of the vacuum which we call electric polarisation and $\vec{E}$ a mathematical artefact found by dividing the force on a test charge by its charge. The case is made for regarding the electric potential ϕ of the electron's electric field as its fundamental property.

The first 5 sections refer to classical physics with charges moving at speeds which are very small compared to the speed of light. This view does not admit the possibility that motion at even moderate speeds causes a change in the energy content of the electrons electric field. However, the equations are not understood in terms of Maxwell's aether or any other ether theory. Towards the end of section 3, the case is presented for the individual existence of the electric fields of individual elementary charged particles. The presence of these fields is said to form a background against which electromagnetic interactions take place. This gives validity to Maxwell's equations in the absence of his aether. When the reader meets the equation $\frac{1}{2} m v^2 = \frac{1}{2} \int \vec{B} \cdot \vec{H} \, d\tau$ in section 4, it will differ from the definition of kinetic energy taught in modern electrodynamics and the reader must bear in mind that it is being used to describe late 19th century thinking.

Section 5 explains the generation of the inertial force as a development of this classical understanding and it is only in section 6 that we meet the idea that motion causes changes in the electric field of the charge. The models of Abraham and Lorentz are reviewed and the problems of taking into account the effect of the Lorentz contraction on the energy content of the electric field discussed.

The term Lorentz contraction is used to denote the real contraction of the electron as described by Lorentz in his book "The theory of Electrons". This is very different from the contraction described by SR and used in RED.

In section 7, the effect of the Lorentz contraction is reexamined and the reader is asked to consider the idea that the contracted electric field is best understood by use of the descriptors $\vec{D}$ and ϕ. The electric field strength is then understood to be $\vec{E} = \nabla\phi$ and is found to disassociated from $\vec{D}$ with the factor $\gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}}$ affecting their co-ordinates differently. The theory presented in this paper is now at variance with modern electrodynamics which gives $\vec{E} = \nabla\phi + \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$ (in Gaussian units) and maintains the relationship $\vec{D} = \varepsilon_0 \vec{E}$. (See the appendix)

3: The electron model

Classical physics sees the electron as a small hard massive ball with electric charge stuck to its surface and somehow filling surrounding space with an electric field $\vec{E}$. It then credits space with the property of being able to be electrically polarised to give a field $\vec{D} = \varepsilon_0 \vec{E}$. When two electrons occupy the same region of space, classical physics asserts that $\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$ where $\vec{E}$ is the electric field present, $\vec{E}_1$ is the electric field which would be present if only the first charge were present and $\vec{E}_2$ is the electric field which would be present if only the second charge were present etc. The fields $\vec{E}_1$, $\vec{E}_2$ etc. are not said to have separate existence, but because of the principle of superposition, the mathematics works as if they did exist separately.

Like Feynman, I do not see that this model can possibly explain the action of electric forces over a distance, but his QED is far harder for nature to operate. I take the view that the known mathematical behaviour of charges is more easily explained by a model in which the electron and other elementary charged particles consist of nothing but a quantity of energy deposited in the form of an electric polarisation of space. When we understand all the properties of "electric energy", we will doubtless discover that the energy content, surface charge and inner radius have stable solutions corresponding to the electron, up quark and down quark.

Since we find that the form of a magnetic field surrounding a current carrying circuit is given by:

$$\vec{B} = \mu_0 \sum_i \frac{q_i}{4 \pi r_i^2} \vec{v}_i \wedge \hat{r}_i$$

We can most easily explain its form by assuming that the individual electric fields of the conduction band electrons have separate existences in the space surrounding the circuit. I take this as prima face evidence that the electric fields of individual charges have separate existence. The equation $\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$ is now interpreted very differently. The fields $\vec{E}_1$ and $\vec{E}_2$ etc. have separate existence and each exerts a force $\vec{F}_i = \vec{E}_i q$ on any charge q. These forces then add vectorially and $\vec{E}$ is a mathematical artefact describing the resultant effect on the charge q and the correct form of the equation for $\vec{E}$ is:

$$q \vec{E} = \sum_i \vec{E}_i q$$

I call this the principle of superimposition. Whereas the principle of superposition is a mathematical theorem, my principle of superimposition is the physical mechanism by which nature operates. Whereas we though of $\vec{E}$ as acting on space causing it to be polarised, now we must see the the individual electric fields as separate polarisations which coexist in space.

The mechanism behind the generation of the force $\vec{F}_i = \vec{E}_i q$ is very simple. The charge q_i is not to be seen as a finite object, but as a space filling field extending towards infinity. The surface of q sits within the polarisation of each electric field and directly experiences the electric force between the pulled apart positive and negative charge of the polarisation. This is a direct local action between the extended electric fields $\vec{E}_i$ and q. This has a very immediate effect on the classical theory of the electron. The model of the metal sphere with a layer of surface charge on it no longer applies. In that model, the surface charge was seen as consisting of elements of charge, each surrounded by a spherically symmetric electric field. In the new model the electric field from a surface element of the charge's surface emerges in only one direction: outward and perpendicular to the surface. There is no interaction between the surface elements of the charge because they do not sit in each other's electric fields.

The electron in now seen as an extended object with field and inner surface. It is primarily a stable structure in which energy is able to deposit itself. However, we cannot adequately account for its behaviour with a three dimensional model. The property of electric potential ϕ which many had though of as a mathematical artefact reveals itself as real in the long forgotten experiments of Reiss and Kohlrausch. A parallel plate capacitor is charged and connected to an electrostatic voltmeter. The distance between the plates is varied and the voltage between the plates is found to be directly proportional to their separation.

If we were to examine a region containing part of the surface of a plate away from the edge, we would see nothing happening as the distance between the plates varied. The surface charge distribution would not be disturbed and the electric field $\vec{E}$ would remain the same. Maxwellian theory has it that work is done polarising the extra volume of space between the plates when they are moved apart and energy adsorbed enabling the plates to to work when they are moved together. The new model no longer admits the notion that the $\vec{E}$ of the equation $\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$ is real and can act on space to polarise it. Only the individual fields $\vec{E}_1$ and $\vec{E}_2$ etc. exist and they are each the result of an individual polarisation of space. The only explanation for the recorded change in potential between the plates is their change in position within the electric fields of the charges on the other plate. We must conclude that the ϕ_i are real and that the electric field must be thought of as a four dimensional entity existing in R3+ϕ space. The diagrams we draw of ϕ against x now take on a new meaning as valid representations of a four dimensional reality.

We have to develop a more holistic concept of the electric field of a charge in which $\vec{D}_i$, $\vec{E}_i$ and ϕ_i are descriptors of the field. In the interactions of macroscopic physics, it is the individual $\vec{D}_i$, $\vec{E}_i$ and ϕ_i which act producing forces $\vec{F}_{i,j}$ upon the individual q_j. One of our tasks is to distinguish which descriptors refer to real properties and which are just mathematical artefacts. Nowhere is this better illustrated than in the way we write the equation $\vec{B} = \mu_0 \sum_i \frac{q_i}{4\pi r_i^2} \vec{v}_i \wedge \hat{r}_i$ and the seven equations we can derive from it.

$$\vec{H}_i = \frac{q_i}{4\pi r_i^2} \vec{v}_i \wedge \hat{r}_i \quad \text{or} \quad \vec{H}_i = v_i \wedge \vec{D}_i \quad \text{or} \quad \vec{H}_i = v_i \wedge \varepsilon_0 \vec{E}_i \quad \text{or} \quad \vec{H}_i = v_i \wedge \varepsilon_0 \nabla\phi_i$$

$$\vec{B} = \sum_i \vec{B}_i \quad \text{or} \quad \vec{B} = \mu_0 \sum_i \vec{H}_i \quad \text{or} \quad \vec{B} = \sum_i \mu_0 \vec{H}_i$$

I would suggest that in a classical interpretation, the equations $\vec{B} = \mu_0 \sum_i \vec{H}_i$ and $\vec{H}_i = v_i \wedge \vec{D}_i$ best describe the processes by which nature operates. It might be helpful if the reader ponders this for a little while and gives each of the eight equations a physical interpretation in terms of the processes by which nature might act.

It never occurred to classical physics that the velocities $\vec{v}_i$ needed a more exact interpretation. Provided: that that the summations are made over the right sets of charges; that for each conduction band electron in a circuit, we include one electron unit of positive charge upon an adjacent lattice ion; that we include the orbital electrons active in ferromagnetic materials in the same way and that all $v_i \lll c$, the $\vec{v}_i$ may be measured with respect to any convenient frame and theory and experiment will agree within the laboratory.

But our quest is to match our mathematics with the processes by which nature operates and that leaves no room for an inexact interpretation of the $\vec{v}_i$. Maxwell had no problem with this and believing the "flux" of electric and magnetic fields to reside in the aether, he assumed his $\vec{v}_i$ were measured with respect to it. This meant that the fields of a charge moving through the aether require a mechanism to continual reform them around the moving charge. The fields to the rear must decay and energy must be moved to the front where it formed into new field. The Poynting vector gave the energy flow and latter led to the concept of electromagnetic momentum. The abandoning of aether theory puts all this firmly in the realms of history, yet it takes a lot to get a physicist to abandon a good equation and the mathematics lives on.

The presence of the individual electric fields of all charges in space is sufficient explanation for the generation of magnetic effects. We simply have to postulate that the effect of movement of electric fields through each other generates magnetic effects. More precisely, the motion of the individual field of the charge q_i (as described by $\vec{D}_i$) against the presence of all the other fields generates a magnetic intensity $\vec{H}_i = v_i \wedge \vec{D}_i$. The magnitude of $\vec{H}_i$ does not depend on the number of other charges in the universe, so as we try to take into account the velocity of q_i relative to each of the other q_j in the universe, we must use an averaging process rather than a summation. We borrow from economics the concept of a weighted average dependent on some property of the electric fields. If the weighting factor is the field strength E, then we may write:

$$\vec{v}_{i,\,locally\,absolute} = \frac{\sum_{j \neq i} (\vec{v}_i - \vec{v}_j)\ |\vec{E}_j|}{\sum_{j \neq i} |\vec{E}_j|}$$

and we might calculate a velocity $\vec{s} = \dfrac{\sum_{j \neq i} \vec{v}_j |\vec{E}_j|}{\sum_{j \neq i} |\vec{E}_j|}$ which is the velocity of the background against which the motion of a charge generates magnetic intensity. A Galilean frame invariant expression for the magnetic intensity then becomes $\vec{H}_i = (\vec{v}_i - \vec{s}) \wedge \vec{D}_i$ where $\vec{v}_i$ and $\vec{s}$ are measured from the same reference frame. I call this "stasis theory". It remains to be determined as to which property nature uses as the weighting factor. Using electric field strength, the vector $\vec{s}$ in a laboratory has components of approximately 16 m/s back along the earth's orbital path plus half the velocity of the lab measured in the ECM frame due to the earth's rotation. If electric potential is the weighting factor, then $\vec{s}$ is determined mainly by the distribution and motion of matter in the galaxy! In a Lorentz-Poincaré[1] world, the real slowing of clocks would provide an experimental tool to investigate the alternatives.

4: Electromagnetic mass and momentum

Classical physics regarded electromagnetic mass and momentum as properties of the electromagnetic field. This interpretation depended on Maxwell's understanding of the nature of the fields within the aether. The action of a magnetic field on a moving electron defies Newton's laws and electromagnetic momentum was cited claiming that its inclusion would restore conservation of linear momentum. However, the theory is deficient in that instead of evaluating volume integrals, it converts them to surface integrals and applies them to a surface containing a system. Examination of the Lorentz and Abraham models shows that the integral must be over a volume from the surface of the electron outwards to infinity and that the main factor in determining its magnitude is the size and shape of the inner surface.

In forming a unified theory, it is better not to regard Newton's laws as fundamental and universal, but to derive them from more fundamental principles for specific situations.

We can come to a more intellectually satisfying understanding of these matters if we start with the assumption that everything is composed of energy and that conservation of energy is the primary law. When an electron is accelerated by an electric field, the fundamental process is one of turning the potential energy it posses by virtue of its geometric position within the field into magnetic energy contained in the magnetic field generated by its motion. Mass and force are concepts derived from human experience and they can be used quite nicely to describe the process, but we must understand that Newton's law $\vec{F} = m\,\vec{a}$ is a generalisation describing the result of myriads of electromagnetic interactions in the form of energy exchanges.

In the classical model of a moving charge, a magnetic field is generated with a total energy content proportional to v^2. It is a simple matter to associate this with the kinetic energy $\frac{1}{2}\,m\,v^2$ and obtain the equation $\frac{1}{2}\,m\,v^2 = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,d\tau$, substitute $\vec{H} = \vec{v} \wedge \vec{D}$, and rotate the triple scalar product to get:

$$\tfrac{1}{2}\,m\,v^2 = \tfrac{1}{2}\int \vec{v}\cdot\vec{D}\wedge\vec{B}\,d\tau \;\Rightarrow\; m\,\vec{v} = \int \vec{D}\wedge\vec{B}\,d\tau$$

But we must ask ourselves what this mathematical derivation means. In a Maxwell-Thompson world consisting of nothing but eddies and flows in the aether, it must surely represent the real nature of momentum

[1] Lorentz-Poincaré world = world in which motion against some background causes Lorentz contraction leading to universal validity of Lorentz transforms.

and the fundamental meaning of mass. Without an aether, it is just an interesting mathematical coincidence which we may get quite excited about, but which nature ignores.

Lorentz used the electromagnetic momentum to derive the inertial force and we can only assume that this was because the calculation omits the inconvenient change in the energy content of the electric field. neither the Abraham, Lorentz or relativistic models adequately deal with this. There are two problems, first the predicted increase in energy content of the electric field is of the same order of magnitude as that of the magnetic field and second, increase in energy content of a volume element depends on the direction of the electric field in it. If the Lorentz contraction is correctly applied, then the energy content of the electric field increases by a factor[2] $\frac{1}{3\gamma} + \frac{2\gamma}{3}$ which does not convert into a nice function of γ and there is no way to match the experimental results $m_l = \gamma^3 m_0$ and $m_t = \gamma m_0$. Just as in thermodynamics, the kinetic energy of a gas is partitioned into thirds, one for each degree of freedom, so the energy content of the electric field behaves as if it is partitioned into $\frac{1}{3}$ in the direction of motion and $\frac{2}{3}$ perpendicular to it and these are effected differently by the Lorentz contraction in Lorentz's theory and by the Lorentz transforms in a relativistic theory. Suitable use of a method allowing a vector product to effect a resolution into the direction perpendicular to the motion eliminates the problem, but such methods at best lack rigor and at worst are a fudge.

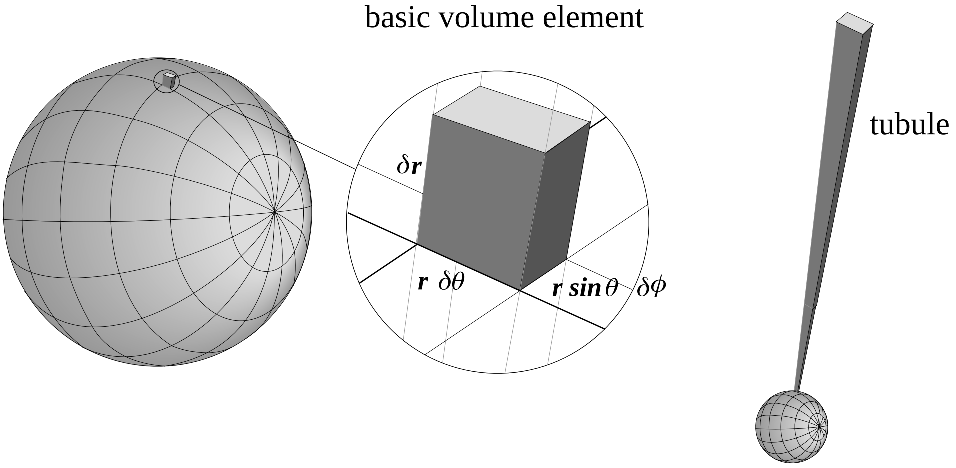
Lorentz discusses Poincaré's internal stresses which seemed to provide a solution at the time. The problem being that energy is supposed to be stored in the internal stresses and released by the Lorentz contraction to do work providing the increase in the energy content of the external electric field. Such energy must be finite, but there is no limit to the energy requires as $v \to c$ so the solution will work only so long as $\gamma <\sim 2$.

Our final model will be derived in two steps. First it will be developed as a classical theory. Then we will look at the derivation of the Lorentz contraction and the nature of fields and see how we might be able interpret them in a way which gives an invariant energy content to the electric field. We have already seen how an electron model with no interaction between its surface elements is possible and this eliminates the possibility in any Poincaré stresses which might affect the energy.

5: A classical theory of inertia

According to classical theory, the moving charge is surrounded by a magnetic field $\vec{B} = \frac{\mu_0 q}{4 \pi r^2} \vec{v} \wedge \hat{r}$. This magnetic field has an energy density $Q_m = \frac{1}{2 \mu_0} B^2 = \frac{\mu_0 q^2}{32 \pi^2 r^4} (\vec{v} \wedge \hat{r})^2$. We consider a surface element of area $\delta A = r^2 \delta\omega = r^2 \sin\theta\, \delta\theta\, \delta\phi$ and the conical volume element which can be constructed outwards from the surface element everywhere parallel to the electric field. Such volume elements will herein after be referred to as "tubules" since they are constructed according to the rule devised by Faraday to define what latter became known as "Faraday tubes".

[2] This is derived in more detail latter.



The basic volume element of the tubule is $\delta\tau = r^2\,\delta\omega\,\delta r$ and the energy content of the magnetic field within the tubule is:

$$\delta\mathscr{E}_m = \frac{\mu_0\,q^2}{32\,\pi^2}\,(\vec{v}\wedge\hat{r})^2\int_{r_0}^{\infty}\frac{1}{r^4}\,r^2\,\delta\omega\,dr = \frac{\mu_0\,q^2}{32\,\pi^2\,r_0}(\vec{v}\wedge\hat{r})^2\,\delta\omega \tag{1}$$

We can now differentiate[3] this with respect to time.

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{\mu_0\,q^2}{32\,\pi^2\,r_0}\,\frac{d}{dt}\left((\vec{v}\wedge\hat{r})\cdot(\vec{v}\wedge\hat{r})\right) = \frac{\mu_0\,q^2}{16\,\pi^2\,r_0}\,(\vec{v}\wedge\hat{r})\cdot(\vec{a}\wedge\hat{r})$$

We assume that this rate of change of energy content is equal to the work done by a force $\delta\vec{F}$. Then

$$\delta\vec{F}\cdot\vec{v} = \frac{\mu_0\,q^2}{16\,\pi^2\,r_0}\,(\vec{v}\wedge\hat{r})\cdot(\vec{a}\wedge\hat{r})$$

This quadruple vector product reduces to the triple scalar product of $\vec{v}$, $\hat{r}$ and $(\vec{a}\wedge\hat{r})$ which is then invariant under cyclic rotation and we may write $\vec{v}\wedge\hat{r}\cdot(\vec{a}\wedge\hat{r}) = \hat{r}\wedge(\vec{a}\wedge\hat{r})\cdot\vec{v}$ giving:

$$\delta\vec{F}\cdot\vec{v} = \frac{\mu_0\,q^2}{16\,\pi^2\,r_0}\,\hat{r}\wedge(\vec{a}\wedge\hat{r})\cdot\vec{v}$$

We have not specified the direction of δF and are free to define its direction as being that of $\hat{r}\wedge(\vec{a}\wedge\hat{r})$. This then allows us to conclude that:

$$\delta\vec{F} = \frac{\mu_0\,q^2}{16\,\pi^2\,r_0}\,\hat{r}\wedge(\vec{a}\wedge\hat{r})$$

In the special case when the acceleration is in the direction of the velocity and $\vec{a} = a_x\,\hat{v}$, the change in the magnetic field only involves a change in magnitude. If we assumed that magnetic flux was being generated within the surface of the charge and emerging from the surface at a velocity $\vec{v}_{flux} = v_{flux}\,\hat{r}$ then according to Faraday's law, an electric field $\vec{E} = \vec{B}\wedge\vec{v}_{flux}$ is induced. This field exerts a force on the surface element of the charge in the direction of $(\vec{v}\wedge\hat{r})\wedge\hat{r}$. That is to say that it is the force resisting $\delta\vec{F}$ and by taking the dot product with $\hat{v}$ we identify the accelerating force and the inertial force resisting it.

We can in this special case calculate a value of $\vec{v}_{flux}$ on the assumption that changes in the magnetic field within the tubule are accomplished by the movement of magnetic energy up and down the tubule. Dividing $\frac{d}{dt}\delta\mathscr{E}_m$ by the product of the energy density and the area of the base of the tubule (much as we would to calculate the velocity of gas in pipe) gives us the required velocity:

[3] See appendix for full working out.

$$\vec{v}_{flux} = \frac{\frac{d}{dt}\delta\mathscr{E}_m}{Q_m\, r_0^2\, \delta\omega} = \frac{\frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\, (\vec{v} \wedge \hat{r}) \cdot (a_x\, \hat{v} \wedge \hat{r})}{\frac{\mu_0\, q^2}{32\,\pi^2\, r_0^4}\, (\vec{v} \wedge \hat{r})^2\, r_0^2\, \delta\omega} = \frac{2\, r_0\, a_x}{v}\, \hat{r}$$

$$\vec{E} = \vec{B} \wedge \frac{2\, r_0\, a_x}{v}\, \hat{r}$$

Since the generation of magnetic flux takes place throughout the thickness of the surface, the induced electric field varies from 0 to $\vec{E}$. The effect of this is quantifiable through energy considerations leading us to introduce a "surface penetration coefficient" of $\frac{1}{2}$ giving:

$$-\delta\vec{F} = \frac{1}{2}\vec{E}\,\delta q = \frac{1}{2}\vec{B} \wedge \vec{v}_{flux}\,\delta q = \frac{1}{2}\,\frac{\mu_0\, q}{4\,\pi\, r^2}\, \left(\vec{v} \wedge \hat{r}\right) \wedge \frac{2\, r_0\, a_x}{v}\hat{r}\, \frac{q}{4\,\pi}\delta\omega$$

(We have already used $\delta\vec{F}$ as the accelerating force, so the resisting force is $-\,\delta\vec{F}$)

By some nimble rearranging

$$-\delta\vec{F} = \frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\, \left(\vec{a} \wedge \hat{r}\right) \wedge \hat{r} \qquad : \vec{a} = a_x\, \hat{v}$$

If we consider the differentiation of $\vec{B}$ in the case of acceleration in any direction,

$$\frac{d}{dt}\vec{B} = \frac{d}{dt}\left(\frac{\mu_0\, q}{4\,\pi\, r^2}\, \vec{v} \wedge \hat{r}\right) = \frac{\mu_0\, q}{4\,\pi\, r^2}\, \vec{a} \wedge \hat{r}$$

In the case of acceleration in the direction of the velocity, we saw that the inertial force results from the action on the charge on the surface element of an electric field induced by the emergence of flux. When the acceleration is an any direction, then the magnetic field undergoes changes in energy content and direction. The required flow of magnetic flux into the tubule now has the directional property of $\frac{d}{dt}\vec{B}$. We know that the equation $-\delta\vec{F} = \frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\, \left(\vec{a} \wedge \hat{r}\right) \wedge \hat{r}$ is correct and that in the case $\vec{a} = a_x\, \hat{v}$ this corresponds to the generation of a force through induction. We are now in a position to remove the restriction and conclude that the process of induction is also responsible for the force in the general situation.

This leads us to an understanding of the nature of magnetic flux in which its "substance" is energy, but its ability to induce an electric field is its flux density. As magnetic flux emerges from (or is adsorbed into) the surface of the charge, the direction of $\frac{d}{dt}\vec{B}$ controls the induction. While this might seem odd, it is an interpretation which enables us get a seamless integration of the process of induction and the conversion of energy between its mechanical and magnetic forms.

The inertial force is given by integrating over the surface of the charge.

$$\vec{F} = -\frac{\mu_0\, q^2}{16\,\pi^2\, r_0} \int \left(\vec{a} \wedge \hat{r}\right) \wedge \hat{r}\; d\omega$$

$$\vec{F} = -\frac{\mu_0\, q^2}{16\,\pi^2\, r_0} \int_0^{2\,\pi} \int_0^{\pi} \left(\vec{a} \wedge \hat{r}\right) \wedge \hat{r}\, sin\,\theta\; d\theta\; d\phi = \frac{\mu_0\, q^2}{16\,\pi^2\, r_0} \begin{pmatrix} -\frac{8\,\pi}{3}a_x \\ -\frac{8\,\pi}{3}a_y \\ -\frac{8\,\pi}{3}a_z \end{pmatrix} = -\frac{\mu_0\, q^2}{6\,\pi^2\, r_0}\vec{a}$$

(This integration is given in full in the appendix)

This takes the form of Newton's second law of motion if $m = \frac{\mu_0\, q^2}{6\,\pi\, r_0}$ and $r = \frac{\mu_0\, q^2}{6\,\pi\, m}$. The energy stored in the electric field of a charge of this size is $\mathscr{E}_e = \frac{q^2}{8\,\pi\,\varepsilon_0\, r_0}$ which gives an inertial mass equal to $\frac{4}{3}$ of the mass equivalent of the electric field calculated from Einstein's $E = m\,c^2$. Note that result comes from the classical limit of $v \ll c$.

6: Previous models

Early experimental data on the variation of mass with speed of beta particles suggested that the mass behaved differently under linear and centripetal acceleration. Two types of mass were identified. Linear acceleration was resisted by "longitudinal mass" and centripetal acceleration by "transfers mass". These were given by $m_l = \gamma^4 m_0$ and $m_t = \gamma^2 m_0$. Abraham showed that the energy in the electric and magnetic fields of a rigid spherical electron with Lorentz contracted field behaved as $\frac{e^2}{8\pi}\left(\frac{3c^2 - v^2}{cv} log \frac{c+v}{c-v} - 2\right)$ and $\frac{e^2}{8\pi}\left(\frac{c^2 + v^2}{cv} log \frac{c+v}{c-v} - 2\right)$. The longitudinal and transfers masses can be derived from these diabolical expressions to give answers, which when expanded as power series in $\frac{v^2}{c^2}$ approximate to $\gamma^4 m_0$, and $\gamma^2 m_0$.

Lorentz preferred a model in which the surface of the electron also suffered a Lorentz contraction. By considering only the magnetic energy and equating that to kinetic energy, he derived $m_l = \gamma^3 m_0$ and $m_t = \gamma m_0$ as an exact result which he also derived by considering the electromagnetic momentum. Although these results contradicted current experimental data, he preferred them on mathematical grounds. In his his 1906 lectures printed in 1915 as "The Theory of Electrons", Lorentz presents Abraham's model in section 26 and discusses his own theory at length towards the end of the book stating why he would have liked to it be correct. By 1915, experimental data on transverse and longitudinal mass confirmed Lorentz's theory over that of Abraham and Lorentz records this in a footnote. He goes to great lengths to avoid a direct integration of the energy content of the Lorentz contracted electric field. Written in the co-ordinates of the stationary system, this is a beast, but after great calculations first to form the integral and then to commence the integration by means of substitution, we arrive at the integrals which we could have simply been written down in the co-ordinates of the moving system.

$$\mathcal{E} = \frac{1}{\gamma} \int_0^{2\pi} \int_0^{\pi} \int_{r_0}^{\infty} \frac{1}{2} \varepsilon_0 \begin{pmatrix} E_x \\ \gamma E_y \\ \gamma E_z \end{pmatrix}^2 r^2 \sin\theta \, dr \, d\theta \, d\phi$$

Writing the result in terms of $\mathcal{E}_0$, the energy content of the electric field when at rest, we get:

$$\mathcal{E} = \left(\frac{1}{3\gamma} + \frac{2\gamma}{3}\right)\mathcal{E}_0$$

The effect of the dot product $\vec{D} \cdot \vec{E}$ is to partition the energy content of the electric field into $\frac{1}{3}$ associated with the component of the electric field in the in the direction of motion and $\frac{2}{3}$ with that perpendicular to it. The Lorentz contraction affects these differently, increasing the energy density of the perpendicular component by a factor γ^2, but leaving the energy density of the parallel component unchanged.

From a classical point of view, the energy content of the magnetic field is real and can be thought of as an additional form of kinetic energy. Expressing the kinetic energy of an electron as the sum of mechanical and electromagnetic parts, Lorentz showed it must be wholly electromagnetic in nature. If we include the increase in the energy content of the electric field, each of the two terms presents a particular problem. The $\frac{1}{3\gamma}\mathcal{E}_0$ will never allow us to obtain expressions for m_l and m_t which are integer powers of γ times the rest mass. The $\frac{2\gamma}{3}\mathcal{E}_0$ will give us twice the required mass. On its own, this would be of no consequence, but the result enables a radius to be calculated for the electron and this must be in reasonable agreement with the radius deduced from radiation scattering. The rest mass of the electron can also be given as a function of the energy content of the electric field giving $m_0 = \frac{4}{3c^2}\mathcal{E}_0$. Einstein's 1905d equates $E = mc^2$ and physicist were left the problem of explaining the extra third. If the increase in the electric energy of the field were taken into account, there would be an extra five thirds to explain away.

Subsequent interpretations of the classical model in the light of Einstein's SR saw the magnetic field as an artefact of the relative velocity of the observer to the electron. With the unwanted $\frac{1}{3\gamma}\mathcal{E}_0$ lost in the calculations, the increase in energy content of the electric field is equal to the energy content of the magnetic field and it was a simple matter to form a Lorentz invariant by subtracting the magnetic energy from the electric energy. Jackson in section 17.5 does this in the context of the Abraham-Lorentz model.

We need to define the boundary line between classical and relativistic physics. Relativistic physicists tend to claim anything involving γ as their exclusive territory. This is quite wrong. The boundary is clearly defined by the claim of Einstein "that the stationary system is superfluous". When Lorentz combined Poisson's and Maxwell's equations to deduce the Lorentz contraction, he was working within the bounds of classical physics. The embryo Lorentz-Poincaré relativity was in part classical and it only starts to cross the boundary at the point where a non absolute synchronisation of remote clocks is admitted and the Lorentz transforms are derived. The Lorentz model of the electron is classical. Relativistic interpretations of the Lorentz model are very different from the original model in that the classical interpretation of the nature of the magnetic field is abandoned and the magnetic field is now seen as an artefact of the observation of the electric field of the electron by an observer moving through its rest frame.

In the model which I will develop, the energy content of the electric field remains constant if the electron's surface and the electric field as described by $\vec{D}$ and ϕ suffer a Lorentz contraction. We need to be a little careful here because we can describe the effects of the Lorentz contraction in terms of a point in the field, or a point in space. For example in Jackson's equation 11.148, we read that "$E_1' = E_1$" which refers to a point within a field whereas in the last paragraph of section 11.10, he says "Along the direction of motion, the field strength is down by a factor of γ^{-2} relative to isotropy, while in transverse directions, it is larger by a factor of γ." When considering a point in space, the combined effect of the Lorentz contraction and the inverse square law dependence of field strength must be taken into account.

Referring to a point within the field, if we take $\vec{E} = \nabla\phi$ instead of $\vec{E} = \frac{1}{\varepsilon_0}\vec{D}$, we find $\vec{E}$ increased by a factor of γ in the direction of motion and $\vec{D}$ larger by a factor of γ in transverse directions. J.H. Jeans in The Mathematical Theory of Electricity and Magnetism, Cambridge 1923 arrives at this result for $\vec{E}$ in section 659. Referring to a point in space, the final paragraph states "the electrostatic forces in S' are $\frac{1}{\kappa}$ (= $\frac{1}{\gamma}$) times those in S as regards their x-components, but κ times those in S with regard their y components." When we take into account the effect of the inverse square law within the Lorentz contracted field, my result corresponds to $E_x' = \frac{1}{\gamma} E_x$ which is Jeans's result. Ironically in the preceding section, Jeans gives Abraham's result and the following Lorentz analysis in terms of electromagnetic momentum, but he never made the connection.

7: The effect as $v \to c$

From experimental evidence we know that as the velocity of a charge approaches the speed of light, it behaves as if it had two different inertial masses. Transverse mass $m_t = \gamma m_0$ and longitudinal mass $m_l = \gamma^3 m_0$. (These terms have fallen out of usage) The reason for this in classical terms is very simple. A moving magnetic field induces an electric field. When $v \ll c$ the electric field induced by the motion of the magnetic field surrounding the charge is very small compared with the electric field of the charge. Only when v approaches the speed of light does it become significant and there is a feedback effect. The two problems in modelling this feedback are first to obtain functions of the form γ^n and second to limit the powers of γ.

We might reasonably expect to be able to deal with the secondary electric field induced by the motion of the magnetic field by using the principle of superposition. This being so we might calculate a secondary magnetic field due to the motion of the secondary electric field. Repeating the process, the electric and magnetic fields become power series of the form $D = D_0\left(1 + \frac{v^2}{c^2} + \left(\frac{v^2}{c^2}\right)^2 + \left(\frac{v^2}{c^2}\right)^3 + \ldots\right)$. This is the expansion of γ^2. The result is to increase $\vec{B}$ and the components of $\vec{D}$ perpendicular to the direction of motion by a factor of γ^2. This would then result in $m_t = \gamma^3 m_0$ and $m_l = \gamma^3\left(1 + \frac{v^2}{c^2}\right)m_0$. As this is not the case, something must be limiting the feedback process.

Lorentz was able to obtain the correct behaviour of the magnetic field by assuming that the charge remained constant as its surface suffered a Lorentz contraction. This allowed hm to use Gauss's equation as a boundary condition and conclude that the field as described by $\vec{D}$ suffered a Lorentz contraction. But he assumed that this also applied to $\vec{E}$ and concluded that the energy content of the electric field changed. Lorentz's electron model goes part way to accounting for the observed effects. To see how we might correct the model, we should start by examining the derivation of the Lorentz contraction[4].

We consider a system of charges in motion through some background against which the motion of electric and magnetic fields generates magnetic and electric fields. We write Poisson's equation $\nabla^2\phi = \frac{\rho}{\varepsilon_0}$ and Maxwell's wave equation $\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi = 0$ expressed in terms of electric potential and combine them to get $\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi = \frac{\rho}{\varepsilon_0}$. For a moving system $\frac{\partial}{\partial t} = \frac{\partial x}{\partial t}\frac{\partial}{\partial x}$ and substitution and expansion of $\nabla^2\phi$ yields:

$$\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2}{\partial x^2}\phi + \frac{\partial^2}{\partial y^2}\phi + \frac{\partial^2}{\partial z^2}\phi = \frac{\rho}{\varepsilon_0}$$

The substitution $x' = \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}}$ restores this to Poisson's equation and we conclude that if the system is in equilibrium when at rest, it will be in equilibrium when in motion if it is contracted by a factor $\sqrt{1 - \frac{v^2}{c^2}}$ in the direction of motion.

However, this a differential equation with a multitude of possible solutions and we must apply boundary conditions. Assuming that Gauss's law still applies we conclude that the total electric flux remains $\int \vec{D} \cdot dA = 4\pi q$. We therefore conclude that the surface of the electron and its electric field as described by $\vec{D}$ are Lorentz contracted.

A problem arises from the fact that the electric field surrounding an electron has three descriptors, $\vec{D}$, $\vec{E}$ and ϕ. It has always been assumed that the relationship $\vec{D} = \varepsilon_0 \vec{E}$ would be maintained. However, if this is the case, since $\vec{E} = \nabla\phi$ our view from the stationary system sees the electric field as described by ϕ elongated and the equipotential surfaces no longer perpendicular to the electric field. Worse still, the surface of the charge no longer coincides with an equipotential surface. Perhaps ϕ is simply a mathematical artefact of no physical significance and these problems are of little consequence. But the derivation of the contraction was in terms of ϕ and although we can write our equations in terms of E, Poisson's equation turns into a first order differential equation while Maxwell's wave equation in $\vec{E}$ is of the second order. The derivation will no longer work. We conclude that the electric field strength must be given by $\vec{E} = \nabla\phi$ and that the relationship $\vec{D} = \varepsilon_0\vec{E}$ no longer holds in the case of motion at near light speeds. This then means that as seen from the stationary system the surface of the electron and its electric field as described by $\vec{D}$ and ϕ are Lorentz contracted. In dielectric theory, some materials are not isotropic and $\vec{D}$ and $\vec{E}$ do not remain parallel. It is found that the energy density of the resulting field is still given by $\frac{1}{2}\vec{D} \cdot \vec{E}$.

We might better understand this if we see invariance of the energy content of the electric field as a boundary condition intimately linked with the use of Gauss's law as a boundary condition.

In the system S moving with the electron, all measuring devices are Lorentz contracted and all clocks run slow. Measurements of the electron and its electric field are the same as they would be in the stationary system if the electron were at rest. The energy density of the electric field is given by $Q_e = \frac{1}{2}\vec{D} \cdot \nabla\phi$ and its

[4] Herein "Lorentz contraction" refers to the real contraction caused by motion though the stationary system.

energy content is $\mathscr{E}_e = \int \frac{1}{2} \vec{D} \cdot \nabla\phi \, d\tau$ in the moving system and $\mathscr{E}_e' = \int \frac{1}{2} \vec{D}' \cdot \nabla\phi' \, d\tau'$ in the stationary system.

$$\vec{D}' = \begin{pmatrix} D_x \\ \gamma D_y \\ \gamma D_z \end{pmatrix} \qquad \nabla\phi' = \begin{pmatrix} \gamma \frac{d\phi}{dx} \\ \frac{d\phi}{dy} \\ \frac{d\phi}{dz} \end{pmatrix} \qquad d\tau' = \frac{1}{\gamma} d\tau$$

And $$\mathscr{E}_e' = \int \frac{1}{2} \vec{D}' \cdot \nabla\phi' \, d\tau' = \int \frac{1}{2} \begin{pmatrix} D_x \\ \gamma D_y \\ \gamma D_z \end{pmatrix} \cdot \begin{pmatrix} \gamma \frac{d\phi}{dx} \\ \frac{d\phi}{dy} \\ \frac{d\phi}{dz} \end{pmatrix} \frac{1}{\gamma} d\tau = \int \frac{1}{2} \vec{D} \cdot \nabla\phi \, d\tau$$

The energy content of the electric field of the electron is invariant. (Readers familiar with the field transforms of SR will find this very hard to accept. The fundamental difference is that SR does not credit the magnetic field with real existence. The derivation above is classical. From the stationary system we see both the moving charge and its moving magnetic field and identify two superposed electric fields, one belonging to the electron and the other induced by the motion of the magnetic field. SR describes the effect of these two fields on a test charge at rest in the stationary system by performing a Lorentz transform of the electric field of the electron from its rest frame to the stationary frame. The two interpretations are different.)

The magnetic field may also be calculated as $\vec{B} = \frac{1}{c^2} \vec{v} \wedge \vec{D}$. With $\vec{v}$ directed along the x axis, only the y and z components of $\vec{D}$ are involved in the vector product and the magnetic field seen from the stationary system is $\vec{B}' = \frac{1}{c^2} \vec{v} \wedge \vec{D}' = \gamma\vec{B}$. It follows that $Q_m' = \gamma^2 Q_m$ and $\mathscr{E}_m' = \gamma \mathscr{E}_m$.

This presents us with a problem. The enhanced magnetic field is seen from the stationary system, but in that system, the spherical symmetry has been broken and the integration is no longer simple. We also have to take into account the fact that the angle of the tubule to the direction of motion appears different. It would be nice to perform the integration in the moving system. To perform the calculations, we translate the problem to an auxiliary system which has the co-ordinates of the moving system, but the magnetic field strength seen in the stationary system. We can then calculate the energy change and the force generated in this system knowing that our final answers must be divided by γ to take into account the change in the size of the volume element.

$$\delta\mathscr{E}_m = \gamma^2 \frac{\mu_0 q^2}{32 \pi^2} (\vec{v} \wedge \hat{r})^2 \int_{r_0}^{\infty} \frac{1}{r^4} r^2 \, \delta\omega \, dr = \gamma^2 \frac{\mu_0 q^2}{32 \pi^2 r_0} (\vec{v} \wedge \hat{r})^2 \, \delta\omega$$

And the rate of change of the energy content of the tubule is:

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{\mu_0 q^2}{32 \pi^2 r_0} \delta\omega \frac{d}{dt}\left(\gamma^2 (\vec{v} \wedge \hat{r}) \cdot (\vec{v} \wedge \hat{r})\right)$$

Knowing in advance that acceleration in the direction of $\vec{v}$ and perpendicular to it have different effects, we associate the γ^2 with one of the velocity terms writing it in a form which differentiates naturally into longitudinal and transverse components.

$$\frac{d}{dt}\delta\mathscr{E}_m = \frac{\delta\mathscr{E}_e}{c^2} \frac{d}{dt}\left(\left((\gamma^2 v) \hat{v} \wedge \hat{r}\right) \cdot (\vec{v} \wedge \hat{r})\right)$$
$$= 2\gamma^2 \frac{\delta\mathscr{E}_e}{c^2} \vec{v} \wedge \hat{r} \cdot \left(\left(\gamma^2 a_x \hat{i} + a_y \hat{j} + a_z \hat{k}\right) \wedge \hat{r}\right)$$

(This differentiation is given in full in the appendix)

Writing $\gamma^2 a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$ as $\vec{a}_\gamma$ and after cyclic rotation of the triple scalar product following the method used above.

$$\frac{d}{dt}\delta\mathcal{E}_m = 2\gamma^2\frac{\delta\mathcal{E}_e}{c^2}\,\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right) \cdot \vec{v}$$

Following the reasoning used above, we might conclude that in the auxiliary system $\vec{F} = 2\frac{\delta\mathcal{E}_e}{c^2}\gamma^2 \int \left(\vec{a}_\gamma \wedge \hat{r}\right) \wedge \hat{r}\; d\omega$. Allowing for the reduced capacity of the volume element, the inertial force experienced in the stationary system is then:

$$\begin{aligned}\vec{F}' &= 2\frac{\delta\mathcal{E}_e}{c^2}\gamma \int \left(\vec{a}_\gamma \wedge \hat{r}\right) \wedge \hat{r}\; d\omega \\ &= -\frac{4\,\mathcal{E}_e}{3\,c^2}\;\gamma\,\vec{a}_\gamma\end{aligned}$$

This is the same result as that derived by Lorentz. We saw in section 4 that the force resulted from the action upon the surface of the charge of the electric field induced by the motion of the magnetic flux as it emerged from or was adsorbed into the surface of the charge. Our result here of the same form, but with the inclusion of the γ factors and we can assume that this interpretation is still valid.

In accelerating high speed charged particles and observing changes in inertial mass, the velocity of our lab through the stationary is very small compared to the velocity of the charge through the lab and we are in effect exerting forces from the stationary system on the moving system of the charge.

8: The factor of $\frac{4}{3}$

Much has been made of the fact that the inertial mass is $\frac{4\,\mathcal{E}_e}{3\,c^2}$ where $\mathcal{E}_e$ is the energy content of the electric field in apparent disagreement with Einstein's $E = m\,c^2$. The author is of the opinion that, when we take into account the different geometry of the fields of electrons and photons, this result is in direct agreement with Einstein's $E = m\,c^2$.

The equation $\vec{B} = \frac{\mu_0\,q}{4\,\pi\,r^2}\,\vec{v} \wedge \hat{r}$ can be squared and rearranged to give an equivalent relationship expressed in terms of energy densities of the electric and magnetic fields: $Q_m = \frac{v^2}{c^2}\,sin^2\theta\;Q_e$. Integration gives the average value of $sin^2\theta$ over the spherical symmetry as $\frac{2}{3}$ and we can write $\mathcal{E}_m = \frac{2\,v^2}{3\,c^2}\,\mathcal{E}_e$. Under linear acceleration:

$$\frac{d}{dt}\mathcal{E}_m = F\,v = \left(m\,a\right)\,v = \frac{d}{dt}\frac{2\,v^2}{3\,c^2}\,\mathcal{E}_e = 2\,v\,\frac{2\,\mathcal{E}_e}{3\,c^2}$$

The inertial mass is then seen to be $2 \times \frac{2}{3}\,\frac{\mathcal{E}_e}{c^2}$ with the 2 coming from the differentiation of v^2 and the $\frac{2}{3}$ from the integration over the spherical symmetry. For a photon, the electric and magnetic fields are perpendicular to the direction of motion and the factor $\frac{2}{3}$ becomes 1. Thus we might expect the mass of a photon to be $2\,\frac{\mathcal{E}_e}{c^2}$ which is simply another way of expressing Einstein's result. Note that Einstein's derivation of $E = m\,c^2$ applies to photons. On their adsorption by an atom, the increase in mass of the atom is given by $E = \delta m\,c^2$ which is correctly written $E_{photon} = \delta m_{atom}\,c^2$. The extension of this to all ponderable matter in the form of $E = m\,c^2$ is unfounded.

[In a later paper of this series, I will show how gravitational and inertia masses of ponderable matter are not strictly speaking equal, but merely proportional. When this analysis is applied to photons, we get a deeper understanding of the relationship between inertia, mass and energy and gravity.]

Appendix

1, The differentiation involved in the result $\frac{d}{dt}\delta\mathcal{E}_m \;=\; \frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\;(\vec{v}\;\wedge\;\hat{r})\;\cdot\;(\vec{a}\;\wedge\;\hat{r})$

$$\frac{d}{dt}((\vec{v}\;\wedge\;\hat{r})\;\cdot\;(\vec{v}\;\wedge\;\hat{r})) \;=\; 2\;(\vec{v}\;\wedge\;\hat{r})\;\cdot\;\frac{d}{dt}(\vec{v}\;\wedge\;\hat{r})$$

Now $\quad \frac{d}{dt}(\vec{v}\;\wedge\;\hat{r}) \;=\; \vec{a}\;\wedge\;\hat{r}\;+\;\vec{v}\;\wedge\;\frac{d\hat{r}}{dt} \quad$ but $\frac{d\hat{r}}{dt} \;=\; 0$

Therefore $\quad \frac{d}{dt}\delta\mathcal{E}_m \;=\; \frac{\mu_0\, q^2}{16\,\pi^2\, r_0}\;(\vec{v}\;\wedge\;\hat{r})\;\cdot\;(\vec{a}\;\wedge\;\hat{r})$

2, The integration involved in obtaining the result $\vec{F} \;=\; 2\frac{\delta\mathcal{E}_e}{c^2}\gamma^2\int\;(\vec{a}_\gamma\;\wedge\;\hat{r})\;\wedge\;\hat{r}\;d\omega \;=\; -\frac{4\,\mathcal{E}_e}{3\,c^2}\;\gamma^2\;\vec{a}_\gamma$:

$$\int_0^{2\pi}\int_0^{\pi}\;(\vec{a}\;\wedge\;\hat{r})\;\wedge\;\hat{r}\,\sin\theta\;d\theta\;d\phi \;=\; \int_0^{2\pi}\int_0^{\pi}\left(\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}\;\wedge\;\begin{pmatrix} \cos\theta \\ \sin\theta\cos\phi \\ \sin\theta\sin\phi \end{pmatrix}\right)\;\wedge\;\begin{pmatrix} \cos\theta \\ \sin\theta\cos\phi \\ \sin\theta\sin\phi \end{pmatrix}\sin\theta\;d\theta\;d\phi$$

$$=\;\int_0^{2\pi}\int_0^{\pi}\begin{pmatrix} a_z\sin^2\theta\cos\theta\sin\phi\;-\;a_x\sin^3\theta\sin^2\phi\;-\;a_x\sin^3\theta\cos^2\phi\;+\;a_y\sin^2\theta\cos\theta\cos\phi \\ a_x\sin^2\theta\cos\theta\cos\phi\;-\;a_y\sin\theta\cos^2\theta\;-\;a_y\sin^3\theta\sin^2\phi\;+\;a_z\sin^3\theta\sin\phi\cos\phi \\ a_y\sin^3\theta\sin\phi\cos\phi\;-\;a_z\sin^3\theta\cos^2\phi\;-\;a_z\sin\theta\cos^2\theta\;+\;a_x\sin^2\theta\cos\theta\sin\phi \end{pmatrix}d\theta\;\;d\phi$$

$$=\;\begin{pmatrix} -\frac{8\,\pi}{3}a_x \\ -\frac{8\,\pi}{3}a_y \\ -\frac{8\,\pi}{3}a_z \end{pmatrix}$$

3, The differentiation involved in the result $\frac{d}{dt}\delta\mathcal{E}_m \;=\; 2\,\gamma^2\frac{\delta\mathcal{E}_e}{c^2}\;\hat{r}\;\wedge\;(\vec{a}_\gamma\;\wedge\;\hat{r})\;\cdot\;\vec{v}$

$$\frac{d}{dt}(((\gamma^2\,v)\;\hat{v}\;\wedge\;\hat{r})\;\cdot\;(\;\vec{v}\;\wedge\;\hat{r})) \;=\; \frac{d}{dt}((\gamma^2\,v)\;\hat{v}\;\wedge\;\hat{r})\;\cdot\;(\;\vec{v}\;\wedge\;\hat{r})\;+\;((\gamma^2\,v)\;\hat{v}\;\wedge\;\hat{r})\;\cdot\;\frac{d}{dt}(\;\vec{v}\;\wedge\;\hat{r})$$

We write $\vec{a} \;=\; \frac{d\vec{v}}{dt}$ as $a_x\,\hat{i}\;+\;a_y\,\hat{j}\;+\;a_z\,\hat{k}$ and note that at this instant $\hat{v} \;=\; \hat{i}$

$$\frac{d}{dt}((\gamma^2\,v)\;\hat{v}\;\wedge\;\hat{r}) \;=\; \frac{d}{dt}\left(\frac{v}{1\;-\;\frac{v^2}{c^2}}\;\hat{v}\;\wedge\;\hat{r}\right) \;=\; \left(\frac{\left(1\;-\;\frac{v^2}{c^2}\right)\,a_x\;-\;v\,\frac{-2\,v}{c^2}\,a_x}{\left(1\;-\;\frac{v^2}{c^2}\right)^2}\,\hat{i}\;+\;\frac{v}{1\;-\;\frac{v^2}{c^2}}\;\frac{d}{dt}\;\hat{v}\right)\;\wedge\;\hat{r}$$

$$\frac{d}{dt}\;\hat{v} \;=\; \frac{d}{dt}\frac{\vec{v}}{v} \;=\; \frac{v\,\vec{a}\;-\;\vec{v}\,(\hat{v}\;\cdot\;\vec{a})}{v^2} \;=\; \frac{a_y\,\hat{j}\;+\;a_z\,\hat{k}}{v}$$

$$\frac{d}{dt}((\gamma^2\,v)\;\hat{v}\;\wedge\;\hat{r}) \;=\; \left(\frac{\left(1\;-\;\frac{v^2}{c^2}\right)\,a_x\;-\;v\,\frac{-2\,v}{c^2}\,a_x}{\left(1\;-\;\frac{v^2}{c^2}\right)^2}\,\hat{i}\;+\;\frac{v}{1\;-\;\frac{v^2}{c^2}}\;\frac{d}{dt}\;\hat{v}\right)\;\wedge\;\hat{r}\;\cdot\;(\;\vec{v}\;\wedge\;\hat{r})\;+\;(\;\gamma^2\,\vec{v}\;\wedge\;\hat{r})\;\cdot\;\left(\vec{a}\;\wedge\;\hat{r}\right)$$

$$=\;\gamma^2\;(\;\vec{v}\;\wedge\;\hat{r})\;\cdot\;\left\{\left(\frac{\left(1\;-\;\frac{v^2}{c^2}\right)\,a_x\;-\;v\,\frac{-2\,v}{c^2}\,a_x}{1\;-\;\frac{v^2}{c^2}}\,\hat{i}\;\;+\;a_y\,\hat{j}\;+\;a_z\,\hat{k}\right)\;\wedge\;\hat{r}\;+\;\;\vec{a}\;\wedge\;\hat{r}\right\}$$

$$=\;\gamma^2\;(\;\vec{v}\;\wedge\;\hat{r})\;\cdot\;\left\{\left(\frac{\left(1\;-\;\frac{v^2}{c^2}\right)\,a_x\;-\;v\,\frac{-2\,v}{c^2}\,a_x}{1\;-\;\frac{v^2}{c^2}}\,\hat{i}\;+\;a_x\,\hat{i}\;+\;2\,a_y\,\hat{j}\;+\;2\,a_z\,\hat{k}\right)\;\wedge\;\hat{r}\right\}$$

$$=\;2\,\gamma^2\,\vec{v}\;\wedge\;\hat{r}\;\cdot\;((\;\gamma^2\,a_x\,\hat{i}\;+\;a_y\,\hat{j}\;+\;a_z\,\hat{k})\;\wedge\;\hat{r})$$

Writing $\gamma^2\,a_x\,\hat{i}\;+\;a_y\,\hat{j}\;+\;a_z\,\hat{k}$ as $\vec{a}_\gamma$ and performing a cyclic rotation of the triple scalar product

$$\frac{d}{dt}\left(\left(\gamma^2 v\right)\hat{v} \wedge \hat{r}\right) = 2\,\gamma^2\,\vec{v} \wedge \hat{r} \cdot \left(\vec{a}_\gamma \wedge \hat{r}\right) = 2\,\gamma^2\,\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right) \cdot \vec{v}$$

4, Validity of $\vec{E} = \nabla\phi$ over $\vec{E} = \nabla\phi + \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$

The term $\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$ in Gaussian units or simply $\frac{\partial\vec{A}}{\partial t}$ in SI units, can be interpreted in two ways. Originally, it referred to the rate of change of the magnetic vector potential ($\vec{A}$) at a point in the aether. Later it referred to the rate of change of $\vec{A}$ at a point in a system. The latter interpretation is followed here. The system is the electron and its fields. Since the form of the magnetic field in the region of the un-accelerated electron, as seen from a co-moving frame, is constant, $\frac{\partial\vec{A}}{\partial t} = 0$ and can be neglected.

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