Inertial Mass as an Electromagnetic Phenomena

Introduction

Following his discovery of the electron, JJ Thompson tried to explain matter as a purely electromagnetic phenomena. A paper by Searle, a colleague, in 1896 looked towards explaining inertia in purely electromagnetic terms. Challenged by this, after several years work, I completed my own theory in the summer of 1997[1]. This explained how a spherical charge of no mass would resist acceleration. This theory was expressed in classical terms and showed how by assuming the substance of the magnetic field to be energy, it was possible to calculate the velocity with which magnetic flux emerged or sank back into the surface of a charge as it was accelerated. That movement generated an electric field which acted on the charge producing an inertial force proportional to the acceleration. The following year, I came across the work of Lorentz and Abraham. Both had derived similar theories, considering electrons moving at high velocity and predicting relativistic increases in mass. My theory was strictly classical, but unlike them, I had described the actual mechanism by which the inertial force is generated.

This paper combines ideas of Lorentz with my previous work to produce a theory explaining how inertial forces are generated and how these forces result in a relativistic increase in inertial mass. Conceptually, it takes us back to 1905 and the evolution of Lorentz-Poincare relativity into Einstein's SR. In this theory, the electron is assumed to have a definite velocity v against some background through which light travels at a constant speed. While personally preferring to identify that background as the presence of the electric fields of all charges[2], this theory is consistent with an understanding of electromagnetism as described by the theory of relativity. In this context, v is the velocity of the charge in the observers inertial frame and the electric field of the charge and its surrounding magnetic field are the observed descriptors of its electromagnetic field as seen by that observer.

SI units are used throughout.

Magnetic energy density flux

If we think of the substance of a magnetic field as being described by its flux density $\vec{B}$ and think in the traditional terms of the movement of magnetic flux, the geometry of the spherical charge presents a problem. If we sum the total flux Φ cutting a half plane through the centre of the charge from its line of motion, we get an infinite answer. If we tried to explain the inertial force resisting acceleration in terms of induction due to the movement of flux into or out of the surface of the charge, the predicted force would be infinite.

If however, we think of the "substance" of a magnetic field as being energy and describe it as an "energy density flux" with an associated property of direction, we find that the field contains a finite amount of "magnetic energy density flux" and the problem becomes solvable. We define a vector quantity:

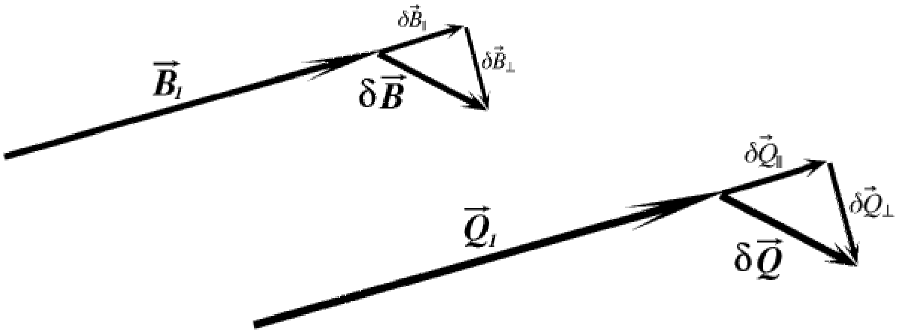
$$\vec{Q}_m = \frac{1}{2\mu_0} B\vec{B} = \frac{\mu_0 q^2 \ v \sin\theta}{32\pi^2 r^4} \vec{v} \wedge \hat{r}$$

This is a vector in the sense that it has magnitude and direction, but it does not form a vector field over addition and scalar multiplication. This leads to problems in differentiating the magnetic energy density vector with respect to time. If we consider a limiting process to find the differential, $\delta\vec{Q}_m$ is a vector with the magnitude $\frac{dQ_m}{dt}\delta t$ and the direction of $\frac{d\vec{B}}{dt}\delta t$. We solve this by drawing the vector triangles representing these

[1] This is published on the internet at <http://users.powernet.co.uk/bearsoft/P1Int.html>

[2] This is based on the proposition that the magnetic intensity, at a point, generated by the motion of an individual charge is due to its velocity relative to each of the other charges in the universe weighted by their relative field strengths at the point. This is then generalised to describe a background against which the motion of electric and magnetic fields generate each other in accordance with Maxwell's laws. This theory is published on the internet at <http://users.powernet.co.uk/bearsoft/StFTi.html>

two limiting processes.



We resolve the incremental vector $\delta\vec{B}$ into two components $\delta\vec{B}_{\parallel}$ and $\beta\vec{B}_{\perp}$ parallel and perpendicular to the magnetic field and do the same for $\delta\vec{Q}$. The action of the flow of magnetic energy into the volume element is now best understood in terms of two components $\delta\vec{Q}_{\parallel}$ and $\delta\vec{Q}_{\perp}$ and their magnitudes $\delta Q_{\parallel}$ and $\delta Q_{\perp}$. The action of the former is to change the energy content of the conic element and that of the latter to change the directional property of its magnetic field. Therefore it is the parallel components which must be compared to determine a scaling factor for the two triangles. These components can be found by differentiating the the scalar definition of magnetic energy density.

$$Q_m = \frac{1}{2\,\mu_0} B^2$$

$$\frac{dQ_m}{dt} = \frac{1}{2\,\mu_0}\frac{d}{dt}B^2 = \frac{1}{\mu_0} B \frac{dB}{dt}$$

$$\delta Q_{\parallel} : \delta B_{\parallel} = \frac{dQ_m}{dt}\delta t : \frac{dB}{dt}\delta t = \frac{1}{\mu_0} B \frac{dB}{dt} : \frac{dB}{dt}$$

So the scaling factor is $\frac{B}{\mu_0}$ and

$$\frac{d}{dt}\vec{Q}_m = \frac{1}{\mu_0} B \frac{d}{dt}\vec{B} \qquad (A)$$

The reader might think this odd preferring $\frac{d}{dt}\left(\frac{1}{2\,\mu_0}B\vec{B}\right) = \left(\frac{1}{2\,\mu_0}B\frac{d\vec{B}}{dt} + \frac{1}{2\,\mu_0}\frac{dB}{dt}\vec{B}\right)$, but it must be remembered that the magnetic energy density does not form a vector field over addition and scalar multiplication rendering the normal methods of differentiation invalid.

A further assumption is needed before we can proceed. That magnetic energy is personal to each charge and can only move parallel to its electric field. It is useful to revise an idea of Faraday's and think of the electric field as divided by surfaces forming tubes whose walls are everywhere parallel to the electric field. Although this idea was superseded by the concept of lines of force, it enables us to picture a conical tube drawn outwards from the rectangular surface element $r_0\, sin\,\theta\;\delta\phi \times r\;\delta\theta$. We can then calculate the total energy within this conical element and consider the movement of magnetic energy into and out of it.

$$\delta\mathscr{E}_m = \int_{r_0}^{\infty} Q_m\, r^2\, sin\,\theta\;\delta\theta\;\delta\phi\; dr$$

Any change in the magnetic field within the conic element must then be associated with a movement of energy out of or into the element $r_0\, sin\,\theta\;\delta\phi \times r\;\delta\theta$ of the charge's surface.

$$\frac{d}{dt}\delta\mathscr{E}_m = \int_{r_0}^{\infty} \frac{d}{dt} Q_m\, r^2\, sin\,\theta\;\delta\theta\;\delta\phi\; dr$$

However, this will not yield the correct result for the resistance to centripetal acceleration. We need to take into account the directional property of the magnetic energy which is being added to or adsorbed from the magnetic field. We therefore need to consider.

$$\frac{d}{dt}\,\delta\vec{\mathscr{E}}_m \;=\; \int_{r_0}^{\infty} \frac{d}{dt}\,\vec{Q}_m\, r^2 \sin\theta\;\delta\theta\;\delta\phi\; dr$$

Relativistic effects

By this term, I mean any modification of an observable quantity by a function of $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.

The mathematical derivation of this factor was first achieved by Lorentz[3] who combined the potential form of the wave equation and Poisson's equation to produce a general equation for the electric fields of a moving system of charges in equilibrium. The substitution $x = \sqrt{1-\frac{v^2}{c^2}}\,x'$ $\quad y = y'$ $\quad z = z'$ again yields Poisson's equation and Lorentz argued that any system held in equilibrium by electrostatic forces when at rest, will when in motion, remain in a state of equilibrium now governed by the same solution in the co-ordinates x', y' and z' resulting in a contraction in the direction of motion. The Lorentz transforms and the time dilation can be deduced from the contraction. Poincare proved that for two inertial systems each in motion through the aether, the result of these effects is to render the Lorentz transforms valid for converting between measurements made in them by a pair of inertial observers provided that their axes were aligned and clocks synchronised in the specified manor. The result is that any attempt to measure the speed of light over a path to a mirror and back will yield the same result. Lorentz postulated that all attempts to measure the speed of a laboratory through the aether will produce null results.

The problem in trying to construct a model of the electron which has the correct inertia at relativistic velocities is to limit the increase. If we expand γ as a series in $\frac{v^2}{c^2}$, it has coefficients $1, \frac{1}{2}, \frac{3}{8}, \frac{5}{16}$ .... ; for γ^3 we get $1, \frac{3}{2}, \frac{15}{8}, \frac{35}{16}$ .... Most models give expressions whose expansions give larger coefficients. Lorentz was able to get his result by making a number of assumptions:

- The electric field and the surface suffer a Lorentz contraction and the field's component perpendicular to the direction of motion increases by a factor γ.
- The magnetic field intensity is consequently increased by the same factor.
- Energy density and electromagnetic momentum consequently increase by a factor γ^2, but the volume element is decreased by γ resulting in increases in the total energy of the magnetic field and the electromagnetic momentum by the required factor γ.
- The total energy in the electric field and body of the electron remains constant.

Abraham had used a transformed field and a spherical surface obtaining horrific expressions. By assuming that the surface of the electron is also contracted, Lorentz was able to do his integration in the spherically symmetric contradicted co-ordinates and just divide by γ getting the correct answer for the energy content of the magnetic field. However he still had a predicted increase in the energy of the electric field which had to be removed. To do this, he made the last assumption which is highly dubious and relies on the invention of internal stresses whose energy content decreases with increased speed. While this might work for low values of γ it becomes ridiculous at high values.

The Lorentz and Abraham electron models are fundamentally different from the Pure Charge model with regard to their internal structure. The Pure Charge model has no internal structure. Its spherical surface is simply the termination of the electric polarisation of space. As such, it is a "raw edge" of charge. It experiences forces from other charges by sitting within the polarised space of their electric fields. The electric field of the Pure Charge extends radially outward from its surface towards infinity. The consequence of this is that an element of the surface of the Pure Charge experiences no force from other surface elements. This completely eliminates the problem of internal energy. It is not a question of the charge possessing energy because it is held together, but the reverse. It is energy which exists, and is indestructible, that coalesces to form the charge.

[3] Lorentz, "The theory of Electrons, ..." 1915
Jeans "The mathematical theory of electricity and magnetism" 1923

Improving on the Abraham Lorentz model

The Lorentz model was deficient only in one respect. Lorentz failed to realise that his derivation of the contraction from equations based on the electric potential ϕ implies that the equipotential surfaces must also suffer a Lorentz contraction. Taking this into consideration, the spherical symmetry of the un-contracted electric field results in the energy content of the field being unchanged by the contraction. All Lorentz's conclusions about the energy content of the magnetic field are correct and his deduction of what he called longitudinal and transverse mass was also correct, even if based on false reasoning about the energy in the charge and its electric field.

By combining the wave equation with Poisson's equation, Lorentz introduces a kind of mutual feedback in which the motion of the magnetic field in turn generates an electric field through induction which modifies the electric field of the charge. Lorentz assumed that Gauss' law would still apply so that the surface integral of the electric flux density would remain constant. Without this assumption an extra factor of γ creeps in giving $m_l = \gamma^4 m_0$ and $m_t = \gamma^2 m_0$. The neatness of Lorentz's model lies in the fact that the modified electric field is both the cause and result of the Lorentz contraction of the charge and its field, but in spite of this, it is unable to account for the change in energy content of the electric field which he effectively choose to ignore.

The normal way of describing the Lorentz contraction is to look at the field at some point in space specified relative to the centre of the charge. This gives a distorted view of the changes in the electric field because the contraction incorporates the effect of the inverse square law maping the point $(\gamma x, y, z)$ onto the point (x, y, z) and comparing the field at (x, y, z) before and after the contraction. It is more informative to map (x, y, z) in S onto $(\frac{1}{\gamma} x, y, z)$ in S' and compare the field descriptors at (x, y, z) before the contraction with those at $(\frac{1}{\gamma} x, y, z)$ after the contraction.

Lorentz thought of the electric field of a charge as consisting of a fixed quantity of electric flux. We need to see the charge and its field in a more holistic way and regard flux density $\vec{D}$, electric intensity $\vec{E}$, potential ϕ, energy density, lines of force and equipotential surfaces as being descriptors of the one thing. When a charge moves with high velocity through the stationary system, it suffers a Lorentz contraction. Using Lorentz's view of lines of force, we can describe a mapping of electric flux density in S onto S' (as described above):

$$\vec{D} = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} \rightarrow \vec{D}' = \begin{pmatrix} D_x \\ \gamma D_y \\ \gamma D_z \end{pmatrix}$$

His mistake was not to realise that the contraction might apply to other descriptors. Since the contraction was derived from equations of potential, it is reasonable to assume that the equipotential surfaces suffer a contraction. This has the effect of increasing the component of $\vec{E}$ in the x direction giving

$$\vec{E} = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \rightarrow \vec{E}' = \begin{pmatrix} \gamma E_x \\ E_y \\ E_z \end{pmatrix}$$

The predicted effect on the energy density $\frac{1}{2} \vec{D} \cdot \vec{E}$ now becomes

$$\frac{1}{2} \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \rightarrow \frac{1}{2} \begin{pmatrix} D_x \\ \gamma D_y \\ \gamma D_z \end{pmatrix} \cdot \begin{pmatrix} \gamma E_x \\ E_y \\ E_z \end{pmatrix} = \gamma \frac{1}{2} \vec{D} \cdot \vec{E}$$

increasing it by a factor of γ rather than γ^2 as in Lorentz'z model. Since the contraction reduces the capacity of each volume element by the same factor, the total energy in the electric field remains constant. Lorentz's analysis of the change in energy content of the magnetic field is now the only change and agrees with the experimental result $m_l = \gamma^3 m_0, \; m_t = \gamma m_0$.

The magnetic field is generated as a result of the motion of the charge through S', the stationary system. It is therefore primarily defined in that system.

$$\vec{H}' = \vec{v} \wedge \vec{D}', \qquad \vec{B}' = \mu_0 \vec{v} \wedge \vec{D}'$$

Relativity predicts that no magnetic field is seen in S, the reference frame of the charge, so we need to create a third "parametric" co-ordinate system S'', in which the magnetic field exists, as a device for integrating over the contracted charge's magnetic field. This parametric system is spacially identical to the view of the charge from S. The electric field of the charge is the same, but the magnetic field takes on the values found in the contracted charge.

$$\vec{H}'' = \vec{H}', \qquad \vec{B}'' = \vec{B}'$$

This changes the pattern of the mapping to:

$$Q_m' \to Q_m'' = Q_m', \qquad \delta\tau' \to \delta\tau'' = \gamma\,\delta\tau', \qquad \mathcal{E}_m' \to \mathcal{E}_m'' = \gamma\,\mathcal{E}_m'$$

The situation with regard to the magnetic field is now that seen by Lorentz when he talks about performing the integration in the contracted co-ordinates.

Generation of the inertial force

We can calculate the changes in energy content of the magnetic field within the conic element by performing the integrals in the parametric system S'', where spherical symmetry prevails and:

$$\vec{B} = \frac{\mu_0 q}{4\pi r^2} \gamma \vec{v} \wedge \hat{r}$$

If the charge experiences an acceleration $\vec{a}$, normal methods of differentiation and some manipulation yields:

$$\frac{d}{dt}\vec{B} = \frac{\mu_0 q}{4\pi r^2} \gamma \; \vec{a}_\gamma \wedge \hat{r} \; : \; \vec{a}_\gamma = \begin{pmatrix} \gamma^2 a_x \\ a_y \\ a_z \end{pmatrix}$$

Substituting in equation (A) gives

$$\begin{aligned} \frac{d}{dt}\vec{Q}_m &= \frac{q}{4\pi r^2} \gamma v \sin\theta \, \frac{\mu_0 q}{4\pi r^2} \gamma \; \vec{a}_\gamma \wedge \hat{r} \\ &= \frac{\mu_0 q^2}{16\pi^2 r^4} \gamma^2 v \sin\theta \, \vec{a}_\gamma \wedge \hat{r} \end{aligned}$$

The rate of change of energy content is accomplished by the generation or adsorption of magnetic energy at the surface of the charge. It is denoted as a vector because of the directional properties it exhibits.

The rate of change of the energy content of a conic element is given by integrating over its volume.

$$\begin{aligned} \frac{d}{dt}\delta\mathcal{E}_m &= \int_{r_0}^{\infty} \frac{d}{dt}\vec{Q}_m \, r^2 \sin\theta \;\; \delta\theta \;\; \delta\phi \;\; dr \\ &= \frac{\mu_0 q^2}{16\pi^2 r_0} \gamma^2 v \sin\theta \; \left|\vec{a}_\gamma \wedge \hat{r}\right| \; \sin\theta \;\; \delta\theta \;\; \delta\phi \end{aligned}$$

Denoting the magnetic energy density at the base of the conic element as $Q_{m,0}$ we can simplify this to:

$$\frac{d}{dt}\,\delta\mathscr{E}_m \;=\; 2\,r_0^3\,Q_{m,0}\,\left|\vec{a}_\gamma \wedge \hat{r}\right|\,\delta\theta\;\;\delta\phi$$

Dividing this by the product of the magnetic energy density at the base of the conic element and the area of its base gives us the velocity v_{flux} with which magnetic energy density flux passes through the surface of the charge.

$$\begin{aligned} v_{flux} \;&=\; \frac{\frac{d}{dt}\,\delta\mathscr{E}_m}{Q_{m,0}\,r_0^2\,sin\,\theta\;\;\delta\theta\;\;\delta\phi} \\ &=\; \frac{2\,r_0^3\,Q_{m,0}\,\left|\vec{a}_\gamma \wedge \hat{r}\right|\,\delta\theta\;\;\delta\phi}{Q_{m,0}\,r_0^2\,sin\,\theta\;\;\delta\theta\;\;\delta\phi} \\ &=\; \frac{2\,r_0}{sin\,\theta}\;\left|\vec{a}_\gamma \wedge \hat{r}\right| \end{aligned}$$

This velocity is in the direction $\hat{r}$ and its cross product with the magnetic induction of the flux (moving into or out of the surface of the charge) gives us the electric field generated. This magnetic induction flux $\vec{B}_{flux}$ takes its direction form $\frac{d}{dt}\,\vec{Q}_m$ and its magnitude from the energy density $\frac{1}{2\,\mu_0}\,B^2$.

$$\vec{B}_{flux} \;=\; \frac{\mu_0\,q}{4\,\pi\,r^2}\,\gamma\,v\,sin\,\theta\,\frac{\vec{a}_\gamma \wedge \hat{r}}{\left|\vec{a}_\gamma \wedge \hat{r}\right|}$$

We can now calculate the electric field generated by the movement of magnetic energy density flux from the surface of the charge into the base of the conic element,

$$\begin{aligned} \vec{E} \;&=\; v_{flux} \wedge \vec{B}_{flux} \\ &=\; \frac{2\,r_0}{sin\,\theta}\;\left|\vec{a}_\gamma \wedge \hat{r}\right|\;\hat{r} \wedge \frac{\mu_0\,q}{4\,\pi\,r_0^2}\,\gamma\,v\,sin\,\theta\,\frac{\vec{a}_\gamma \wedge \hat{r}}{\left|\vec{a}_\gamma \wedge \hat{r}\right|} \\ &=\; \frac{2\,\mu_0\,q}{4\,\pi\,r_0}\,\gamma\,v\;\;\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right) \end{aligned}$$

The surface of the charge has a finite thickness and the magnetic field strength increases linearly as we move from the inner face to the outer face of the surface. This variation does not affect the calculation of the velocity of the flux within the surface, but it does affect the strength of the electric field generated by its motion and we must introduce a factor of $\frac{1}{2}$ called the "surface penetration coefficient" which averages the electric field over the thickness of the surface. The element of charge at the base of the conic element is

$$\delta q \;=\; \frac{sin\,\theta\;\;\delta\theta\;\;\delta\phi}{4\,\pi}\,q$$

The force on the element of charge is

$$\delta\vec{F} \;=\; \frac{1}{2}\,\frac{2\,\mu_0\,q}{4\,\pi\,r_0}\,\gamma\,v\;\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right)\,\frac{sin\,\theta\;\;\delta\theta\;\;\delta\phi}{4\,\pi}\,q$$

giving

$$\begin{aligned} \vec{F} \;&=\; \frac{\mu_0\,q^2}{16\,\pi^2\,r_0}\,\gamma\int_0^{2\,\pi}\!\!\int_0^{\pi}\;\hat{r} \wedge \left(\vec{a}_\gamma \wedge \hat{r}\right)\,sin\,\theta\;\;d\theta\;\;d\phi \\ &=\; -\frac{\mu_0\,q^2}{6\,\pi\,r_0}\,\gamma\begin{pmatrix} \gamma^2 a_x \\ a_y \\ a_z \end{pmatrix} \end{aligned}$$

This is the result derived by Lorentz and now linked with an actual causal mechanism for generating the inertial Force. We can identify this with the relativistic form of Newton's 2nd law:

$$F \;=\; \frac{d}{dt}\;m\,\vec{v}\;\;:\;\;m \;=\; \gamma\,m_0\;\;:\;\;m_0 \;=\; \frac{\mu_0\,q^2}{6\,\pi\,r_0}$$

Analysis of the implications

The first comment to make is that the result allows us to calculate a radius for the electron from its known inertial mass. This radius can then be used to work out a corresponding electrical energy content:

$$\mathscr{E}_e = \frac{q^2}{8\,\pi\,\varepsilon_0\,r_0}$$

If we take the inertial mass corresponding to inertial force and multiply by c^2 to give the energy content of the electron according to Einstein, we get $\mathscr{E}_e = \frac{q^2}{6\,\pi\,\varepsilon_0\,r_0}$ which is greater by a factor of $\frac{4}{3}$ than his prediction. We should not be surprised by the factor of $\frac{4}{3}$ because it arises from the the spherical geometry of the charge and its field. The early attempts to understand the nature of of inertia of an electromagnetic field were based on the "Poynting vector" for momentum which in S.I. units is defined as:

$$\vec{P} = \vec{D} \wedge \vec{B}$$

The concept comes from a time when theorists were trying to reconcile the action of magnetic forces on charges with Newton's conservation of momentum. It is interesting to note that the result we have obtained above can be rearranged as:

$$\begin{aligned}\vec{F} &= \frac{1}{\gamma}\int_0^{2\pi}\int_0^{\pi}\int_{r_0}^{\infty}\frac{q}{4\,\pi\,r^2}\gamma\;\hat{r}\wedge\frac{\mu_0\,q}{4\,\pi\,r^2}\gamma\left(\vec{a}_\gamma\wedge\hat{r}\right)r^2\sin\theta\;d\theta\;d\phi\\ &= \frac{1}{\gamma}\int\vec{D}\wedge\frac{d}{dt}\vec{B}\,d\tau\end{aligned}$$

which we could interpret as *force = Integral of rate of change of electromagnetic momentum over volume*. The concept of Poynting's vector lies back in Maxwell's understanding of the aether as the seat of electromagnetic interactions. The lines of force of the magnetic field of the charge were thought to be stationary in the aether. As the charge moved, energy had to be transferred from the rear to the front of the charge. Poynting found two vectors $\vec{S}$ and $\vec{P}$ representing the energy flow and the momentum associated with it. If we examine the direction of the Poynting vectors, we find that they lie in lines of longitude to the direction of motion. The integration of them over the volume has an averaging effect. If we were to perform the same integration of the fields of a photon which are all perpendicular to the direction of motion, then a factor of 2 rather than $\frac{4}{3}$ results.

The behaviour of γ throughout the calculation is of interest. From the analysis that $\frac{d}{dt}\vec{Q}_m = \frac{1}{\mu_0}B\frac{d\vec{B}}{dt}$, and the differentiation of $\vec{B}$ we get:

$$\frac{d}{dt}\vec{B} = \frac{\mu_0\,q}{4\,\pi\,r^2}\frac{d}{dt}\left(\gamma\,\vec{v}\wedge\hat{r}\right)$$

which is a product and gives

$$\frac{d}{dt}\left(\gamma\,\vec{v}\wedge\hat{r}\right) = \frac{d\gamma}{dt}\left(\vec{v}\wedge\hat{r}\right) + \gamma\left(\vec{a}\wedge\hat{r}\right) = \gamma\begin{pmatrix}\gamma^2 a_x\\ a_y\\ a_z\end{pmatrix}$$

Now the effect of some other power $n > 0$ of gamma at this point would be:

$$\frac{d}{dt}\left(\gamma^n\,\vec{v}\wedge\hat{r}\right) = \frac{d\gamma^n}{dt}\left(\vec{v}\wedge\hat{r}\right) + \gamma\left(\vec{a}\wedge\hat{r}\right) = \gamma^n\begin{pmatrix}n\,\gamma^2 a_x\\ a_y\\ a_z\end{pmatrix}$$

The presence of n in the x component of the vector makes the final answer incompatible with energy considerations for any value of n other than 1. We are thus confirmed in the decision to map $\gamma\vec{B}'$ of the contracted charge onto $\gamma\vec{B}''$ of the third system.

When the velocity of the magnetic flux emerging from the surface of the charge is calculated, apart from the γ^2 in the x component of $\vec{a}_\gamma$, the γ s cancel out and it is not until we substitute for the magnitude of the magnetic induction B that a factor of γ reappears to give a force equal to the rate of change of momentum.

We need now to turn to the teaching of SR on the subject. Namely that the electric and magnetic fields of a charge exist as a single entity and that different observers see them differently according to the velocity of the charge through their frame. If we take the view that nature behaves in accordance with Einstein's special theory of relativity as a consequence of causal effects described by Lorentz - Poincare relativity, then the moving observers do not necessarily observe that which is actually there. Thus an observer in whose frame an electron is stationary will see only its electric field and an observer through whose frame the electron moves with moderate velocity v will see the electric field and a magnetic field given $\vec{B} = \frac{1}{c^2}\vec{v} \wedge \vec{E}$. Both observers can explain what happens as seen from their frame using the same laws of physics, but only in the stationary frame would we actually see the true causal mechanism. We can interpret this view as follows. The actual magnetic field generated by the charge is that seen from the stationary frame. Unable to identify the stationary frame, we in our laboratory frame measure electric and magnetic fields which are Lorentz transforms of the real fields in the stationary system. The interactions between the charge and its fields which we observe are of the same form as those which actually take place in the stationary frame.

I take the observation of phenomena associated with binary stars as prima facie evidence that light travels through every region of space at the same speed. The SR explanation that light travels at the same speed in every reference frame is an insufficient explanation and I interpret the observation of these phenomena as proof of the existence of a background through which light travels at a constant speed. That is not to say that I accept any of the past or current ether theories. The nature of the background and the velocity of our laboratory through it remains a matter for speculation. The fact that that speculation is as yet unresolved does not prevent our reference to "the stationary system" in formulating theories, just as Einstein in his 1905 derivation of the Lorentz transforms used that concept.